

## Annex B: The Fiscal Feedbacks Model

### 1. Introduction

This Annex outlines the model used by the Council to simulate the effects of alternative assumptions for economic growth and paths for discretionary fiscal adjustments. The Council stresses that it is not an alternative projection model to the one used by the Department of Finance; it is designed to exactly reproduce the Government's fiscal projections under their assumptions for growth and discretionary fiscal adjustments. The purpose of the model is to simulate the effects of alternative assumed paths for growth and discretionary adjustments (among other variables) on the paths for the Government's central forecasts for nominal GDP and the key fiscal aggregates. All changes are thus relative to the Government's baseline. The model is used in the report to simulate alternative scenarios and for the generation of the fiscal fan charts given a stochastic path for nominal GDP.

The model takes into account the two-way relationship between nominal GDP and the primary deficit: the primary deficit affects nominal GDP through multiplier effects; and the level of nominal GDP also affects the size of the primary deficit through automatic stabiliser effects. Thus, nominal GDP and the primary deficit are solved for simultaneously. The basic version of the model assumes that nominal GDP depends on the level of the primary deficit (therefore, permanent changes in the primary deficit have permanent effects on nominal GDP). As shown in Section 5, the model can be adapted for alternative dynamic assumptions for fiscal multipliers, and thus permanent year-specific changes to the primary deficit can have varying effects on nominal GDP over time. However, given the current paucity of knowledge on Irish fiscal multipliers, for the simulations in this report we assume that the level of nominal GDP depends on the level of the primary deficit with a multiplier of 0.5. This is consistent with the assumptions on multiplier effects used by the Department of Finance.

Section 2 shows how the model is solved for nominal GDP, the primary deficit (as a share of potential output), the total deficit and the debt. Section 3 considers the effects on key aggregates of alternative growth assumptions and alternative paths for discretionary fiscal adjustment, and thus how the model can be used for simulating deviations from the Government's baseline. Section 4 extends the basic framework to identify the required additional discretionary adjustments that would be required to meet the Government's fixed deficit targets for any values of the exogenous variables. This shows how the required adjustments are determined endogenously for any given

ultimate total deficit targets. Finally, Section 5 extends the analysis to show how alternative dynamic multiplier assumptions can be accommodated.

## 2. Basic Simulation Model

### Model Notation:

$Y$  = Nominal GDP

$Y^*$  = Nominal potential GDP

$Y_0$  = Nominal GDP at a zero primary deficit

$\frac{Y - Y^*}{Y^*}$  = Output gap

$pdef$  = Nominal primary deficit

$pdef^*$  = Nominal structural primary deficit (equals the nominal cyclically adjusted primary deficit minus one-off measures that increase the deficit) =  $pdef^{CA} - v$

$pdef^{CA}$  = Nominal cyclically adjusted primary deficit =  $pdef + b(Y - Y^*)$

$v$  = One-off measures that increase the primary deficit

$def$  = Total nominal deficit (equals nominal primary deficit plus interest payments)

$def^*$  = Total nominal structural deficit (equals nominal structural primary deficit plus interest payments)

$D$  = Total government debt

$sfa$  = Stock-flow adjustment<sup>60</sup>

$m$  = deficit multiplier

$b$  = Automatic stabiliser coefficient

$a$  = Additional discretionary adjustments

$a^R$  = Required additional discretionary adjustments to meet target for total deficit as a share of nominal GDP

$\left(\frac{def}{Y}\right)^T$  = Target for total deficit as a share of nominal GDP

The key behavioural equations are an equation for the output gap and an equation for the primary deficit (as a share of nominal potential GDP). Equation (1) shows that the output gap is the sum of the output gap at a zero primary deficit and a term that depends on the size of the primary deficit (as a share of nominal potential GDP). The coefficient on the primary deficit variable is the deficit multiplier,  $m$ . (In Section 5, alternative assumptions for the dynamic relationship between the primary deficit and the output gap are considered.)

$$\frac{Y-Y^*}{Y^*} = \frac{Y_0-Y^*}{Y^*} + m \left( \frac{pdef}{Y^*} \right). \quad (1)$$

Equation (2) shows that the primary deficit as a share of potential GDP is the structural primary deficit (also as a share of potential GDP) less an adjustment that depends on the size of the output gap and any one-off adjustments,  $v$ . The parameter  $b$ , which we term the automatic stabiliser

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<sup>60</sup> Stock-flow adjustments are defined as the difference between the annual change in the gross debt and the budget deficit (see Weber, 2012). Such adjustments can arise for various reasons, including: (i) valuation effects (e.g. the impact of exchange rate changes on the domestic currency value of foreign-currency denominated debt); (ii) time of recording effects (deficits are based on accrual accounting while the change in debt is based on cash flows); and (iii) “below the line” transactions such as privatisations of state assets and transactions in state-held financial assets (Weber, 2012).

coefficient, determines how the primary deficit deviates from the cyclically adjusted primary deficit,  $pdef^{CA}$ , when there is a positive or negative output gap. The structural primary deficit is the cyclically adjusted primary deficit less any one-off adjustments.

$$\frac{pdef}{Y^*} = \frac{pdef^*}{Y^*} - b \left( \frac{Y - Y^*}{Y^*} \right) + \frac{v}{Y^*}. \quad (2)$$

Note that (2) takes the form used by the Department of Finance to estimate the structural primary deficit. Following the EC's methodology, a value of 0.4 is used by the Department of Finance for the parameter  $b$ . Thus, choosing this value for  $b$  in our simulations ensures consistency with the Department's projections.

Substituting (2) into (1) yields a reduced-form equation for the output gap.

$$\frac{Y - Y^*}{Y^*} = \left( \frac{1}{1 + mb} \right) \left( \frac{Y_0 - Y^*}{Y^*} \right) + \left( \frac{m}{1 + mb} \right) \left( \frac{pdef^* + v}{Y^*} \right). \quad (3)$$

Next, substituting (3) into (2) yields a reduced-form equation for the primary deficit as a share of nominal potential GDP.

$$\frac{pdef}{Y^*} = \left( \frac{1}{1 + mb} \right) \left( \frac{pdef^* + v}{Y^*} \right) - \left( \frac{b}{1 + mb} \right) \left( \frac{Y_0 - Y^*}{Y^*} \right). \quad (4)$$

Up to this point, we have focused on the primary deficit. To obtain the total deficit we need to determine the path of nominal debt. The total deficit,  $def$ , is equal to the primary deficit plus interest expenditure (an accounting identity). We assume the path of the average interest rate on outstanding debt,  $D_{-1}$  is given. Thus the model does not allow for an endogenous determination of the interest rate based on the evolving stock of debt (and thus creditworthiness). The total deficit is then calculated as:

$$def = pdef + iD_{-1}. \quad (5)$$

Next, we allow the total debt to evolve according to a stock-flow relationship:

$$\Delta D = (1 + i)D_{-1} + pdef + sfa. \quad (6)$$

Note that with a given starting value of the lagged debt, equation (6) allows us to identify the path of the debt for any given path of the primary deficit (assuming given paths for the nominal interest rate and stock-flow adjustments). We thus have a four equation system, with the output gap, the primary deficit as a share of potential GDP, the total deficit and the change in the nominal debt as the four endogenous variables. Note that solving for the output gap also identifies the equilibrium level of nominal GDP given the exogenous path for potential GDP. The various endogenous variables can be expressed as shares of nominal GDP or nominal potential GDP as is convenient in any given application.

### 3. Simulating Alternative Scenarios

#### 3.1 Alternative Growth Scenarios

The value of  $Y_0$  is implied in the model based on values for nominal GDP, nominal potential GDP and the nominal primary deficit (and will depend on the chosen value of the deficit multiplier,  $m$ , and the automatic stabiliser coefficient,  $b$ ). Rearranging (3), we obtain an expression for  $Y_0$ ,

$$Y_0 = (1 + mb)Y - mbY^* - m(pdef^* + v). \quad (7)$$

Any assumed path for nominal GDP can be substituted into (7) to obtain the implied path for  $Y_0$ , given the paths for potential nominal GDP and the cyclically adjusted primary deficit. This implied path can then be substituted into (3) and (4) to obtain the simulated paths for the output gap and the primary deficit as a share of potential GDP. Lastly, the paths for the total deficit and the debt are determined as before using (5) and (6).

#### 3.2 Simulating Alternative Fiscal Adjustment Paths

Additional discretionary fiscal adjustments,  $a$ , are assumed to lead to reductions in the structural primary balance,  $pdef^*$ . To allow for such additional adjustments, we can rewrite (4) as:

$$\frac{pdef}{Y^*} = \left( \frac{1}{1+mb} \right) \left( \frac{pdef^* + v - a}{Y^*} \right) - \left( \frac{b}{1+mb} \right) \left( \frac{Y_0 - Y^*}{Y^*} \right). \quad (4')$$

The effects of alternative time paths for the structural primary balance on the output gap (and thus actual nominal GDP given the time path of potential output) and the actual primary balance as a share of potential output are determined using (3) and (4). Once again, the paths of the total deficit and the debt are determined using (5) and (6).

#### 4. Required Additional Discretionary Adjustments to Meet Given Deficit Targets

The model can also be used to identify additional discretionary adjustments,  $a^R$ , that would be required to achieve a given target for the total deficit as a share of GDP,  $\left(\frac{def}{Y}\right)^T$ . (Such deficit to GDP targets are the basic targets in the Government's medium-term fiscal programme, with an objective of reaching a deficit below 3 per cent of GDP by 2015.) Rearranging (4), we obtain:

$$a = pdef^* + v - (1 + mb)pdef - b\left(\frac{Y_0 - Y^*}{Y^*}\right)Y^*. \quad (8)$$

Noting that  $pdef$  is equal to  $def - iD_{-1}$ , we can identify the required additional discretionary adjustments to meet any given total deficit target as:

$$a^R = pdef^* + v - (1 + mb)\left(\left(\frac{def}{Y}\right)^T Y - iD_{-1}\right) - b\left(\frac{Y_0 - Y^*}{Y^*}\right)Y^*. \quad (9)$$

#### 5. Alternative Dynamic Multiplier Assumptions

The basic version of the model assumes that it is the size of the primary deficit that matters for actual GDP (see equation (1)). However, the model can be adapted to allow for alternative dynamic assumptions for the multiplier. Consider, for example, the case where it is only the change in the current period primary deficit that impacts on current GDP, i.e. the effects of fiscal policy on the economy only last for one period. The output equation is then,

$$\frac{Y - Y^*}{Y^*} = \frac{Y_0 - Y^*}{Y^*} + m\left(\frac{\Delta pdef}{Y^*}\right). \quad (10)$$

Note that we now define  $Y_0$  as the level of nominal GDP when the change in the primary deficit is equal to zero.

We can rewrite (2) as:

$$\frac{\Delta pdef + pdef_{-1}}{Y^*} = \frac{pdef^* + v}{Y^*} - b\left(\frac{Y - Y^*}{Y^*}\right). \quad (11)$$

Solving this pair of simultaneous equations yields:

$$\frac{Y - Y^*}{Y^*} = \left(\frac{1}{1 + mb}\right)\left(\frac{Y_0 - Y^*}{Y^*}\right) + \left(\frac{m}{1 + mb}\right)\left(\frac{pdef^* + v - pdef_{-1}}{Y^*}\right), \quad (12)$$

and

$$\frac{pdef}{Y^*} = \left(\frac{1}{1+mb}\right) \left(\frac{pdef^*+v}{Y^*}\right) + \left(\frac{mb}{1+mb}\right) \left(\frac{pdef_{-1}}{Y^*}\right) - \left(\frac{b}{1+mb}\right) \left(\frac{Y_0-Y^*}{Y^*}\right). \quad (13)$$

The foregoing case assumes that only the current period change in the primary deficit affects current nominal GDP. We can relax this assumption by instead assuming that past changes in the primary deficit affect current output, but allow year-specific multipliers based on when the change in the primary deficit occurred. This is the most general form and allows for any time path for the multiplier effects. Letting the impact of a change in the primary deficit impact future output for  $n+1$  years, we can rewrite equation (1) as:

$$\frac{Y-Y^*}{Y^*} = \frac{Y_0-Y^*}{Y^*} + \left(\frac{m\Delta pdef + \sum_{i=1}^n m_{-i}\Delta pdef_{-i}}{Y^*}\right). \quad (14)$$

Using (11) to eliminate the current change in the primary deficit,  $\Delta pdef$ , the reduced form equations for the output gap and the primary deficit as a share of nominal potential GDP are then:

$$\frac{Y-Y^*}{Y^*} = \left(\frac{1}{1+mb}\right) \left(\frac{Y_0-Y^*}{Y^*}\right) + \left(\frac{m}{1+mb}\right) \left(\frac{pdef^*+v-pdef_{-1}}{Y^*}\right) + \left(\frac{1}{1+mb}\right) \left(\frac{\sum_{i=1}^n m_{-i}\Delta pdef_{-i}}{Y^*}\right), \quad (15)$$

and

$$\frac{pdef}{Y^*} = \left(\frac{1}{1+mb}\right) \left(\frac{pdef^*+v}{Y^*}\right) + \left(\frac{mb}{1+mb}\right) \left(\frac{pdef_{-1}}{Y^*}\right) - \left(\frac{b}{1+mb}\right) \left(\frac{\sum_{i=1}^n m_{-i}\Delta pdef_{-i}}{Y^*}\right) - \left(\frac{b}{1+mb}\right) \left(\frac{Y_0-Y^*}{Y^*}\right). \quad (16)$$