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BVARX modeling of the Spanish economy

Abstract

The purpose of the model is the generation of forecasts for the main macroeconomic aggregates that can be easily conditioned on scenarios for the exogenous variables. The operation of the model is closely linked to real-time forecasts produced using dynamic factor models and to the generation of stochastic macroeconomic scenarios that can be plugged in other models as a conditioning input (e.g. satellite models for budgetary projections). In this way the model can be used to check the consistency and reliability of in-house as well as external forecasts.

Written by Ángel Cuevas[T] and Enrique M. Quilis[T]

Revised by Carlos Cuerpo and Gabriel Pérez-Quirós
Approved by José Marín

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[T] Spanish Independent Authority for Fiscal Responsibility

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1 Introduction

Vector of Autoregressions (VAR) models are widely used in empirical macroeconomics because they are simple, easy to estimate and provide a compact and useful representation of the joint behavior of a set of economic time series.

We have used the basic VAR framework enhanced in two dimensions. In the first place, our model incorporates Bayesian features that overcome the overparameterization risk that arise in medium-scale VAR models (the B in BVARX), providing at the same time a way to tailor the model to specific statistical properties of the data.

Secondly, we have included the exogenous variables as a set of conditioning forcing variables (the X in BVARX). This representation produces a simple and neutral way to introduce scenarios about external variables, reducing at the same time the number of required parameters, containing the overparameterization risk previously commented.

In accordance with EU Regulation 473/2013, euro area Member States should have in place independent bodies which produce or endorse macroeconomic forecasts underpinning national medium-term fiscal plans. At national level, the Spanish Independent Fiscal Authority (AIReF), in the exercise of its mandate (in accordance with Organic Law 6/2013), is in charge of endorsing the government medium-term macroeconomic and fiscal projections. In the context of the Draft Budget Law and the Stability and Convergence Programme, the government projects the main macro variables with a time horizon of one to four years ahead. These projections are based on a scenario for some external variables (e.g. non-domestic GDP) that conditions the behavior of the Spanish macroeconomic variables. The BVARX model is an essential component of the toolkit developed by AIReF to perform this assessment.

From an operational perspective, we have developed a suite of MATLAB functions to have full control of tasks of calibration, prior specification, estimation and forecasting. In this way, we can easily integrate the BVARX model with other models (e.g. real-time forecasts based on dynamic factor models or satellite BVAR models) and the existing databases, see Quilis (2015a).

The structure of the paper is as follows. In the second section we briefly describe BVARX models with a prior structure based on the stylized features of most macroeconomic time series. Data and pre-processing issues are presented in the third section. In the fourth section we present the main results of the estimation of the BVARX model and its forecasting performance. We conclude in section five.

2 BVAR and BVARX models

From a technical point of view, BVARX models can be considered as an extension of the standard BVAR models that incorporates a set of additional forcing or external variables (the X part of BVARX). This addition has an important effect on the use of the model as a forecasting tool but it changes very little the Bayesian elements of the model. The structure of this section takes into account this differential impact, explaining fist the main features of BVAR models and expanding them to include a set of exogenous variables.

2.1 BVAR Models

Let $Z_t=(z_1, z_2, .., z_k)_t'$ be a vector of observations on k variables at time t, with $t=1..n$. Also, let Z_t evolve following a vector autoregressive model (VAR) of order p, if it can be expressed in the following way:

[2.1] $$Z_t = c + \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \ldots + \Phi_p Z_{t-p} + U_t$$

where c is a vector of k constant terms and Φ_h, $h=1,\ldots,p$, are kxk matrices. The term U_t represents a vector of zero-mean Gaussian innovations, which are serially uncorrelated and with constant variance-covariance matrix. Then, it is assumed:

[2.2] $$U_t : k x 1 \sim N(0, \Sigma)$$

VAR models are very general structures and, depending on the nature of the matrices Φ_h and Σ, diverse particular cases arise. See Tiao *et al*. (1979), Sargent (1979), Sims (1980), Tiao and Box (1981), Lütkepohl (1991), Tiao (2001), Stock and Watson (2001), Reinsel (2003), Enders (2010) and Karlsson (2012) for an in-depth review of these models.

VAR models are prone to overparameterization and, as a consequence, to overadjustment, imprecise estimation and poor forecasting records. With the aim of solving these problems, Litterman (1984a, 1984b, 1986), Doan *et al.* (1984) and Todd (1984, 1988) propose imposing probabilistic constraints, oriented towards shrinking the size of the parametric space and, as a result, lessening the above mentioned problems. These restrictions are amenable to a Bayesian interpretation and may include quite different structures, depending on the non-sample information that the analyst wants to incorporate in the model. Due to its origin, this prior if often named as 'Minnesota prior' or 'Litterman prior'. This prior has been generalized or modified to consider a wide variety of specifications, see Kadiyala and Karlsson (2007) and Karlsson (2012).

To represent the prior structure, we modify the notation employed in equation [2.1]. Now, the i-th equation of a VAR(p) is:

[2.3] $$Z_i = \begin{pmatrix} i_{n-p} & Z_{1(1)} & \cdots & Z_{k(1)} & \cdots\cdots & Z_{1(p)} & \cdots & Z_{k(p)} \end{pmatrix} \begin{pmatrix} c_i \\ \phi_{i,1,1} \\ \vdots \\ \phi_{i,k,1} \\ \vdots \\ \phi_{i,1,p} \\ \vdots \\ \phi_{i,k,p} \end{pmatrix} + U_i = x\beta_i + U_i$$

being $Z_{j(h)}$ the vector containing *(n-p)* observations of the *j-th* series lagged by *h* periods. Note that the vector of regressors *x* is the same for all the equations, so that the simultaneous consideration of all the *k* equations integrating the VAR gives rise to the following expression:

[2.4] $$Z = (I_k \otimes x)\beta + U = X\beta + U$$

being $\otimes$ the tensor product. Of course, vector β is related to the Φ matrices according to:

[2.5] $$\beta = vec(\Phi') \quad \text{with} \quad \Phi = (c \quad \Phi_1 \quad \cdots \quad \Phi_p)$$

Consequently, the variance-covariance matrix of *U* is:

[2.6] $$\Sigma_U = \Sigma \otimes I_{n-p} \quad \text{with} \quad \Sigma = \{\sigma_{i,j} \quad i, j = 1..k\}$$

Model [2.4] has a similar aspect to standard linear model of the regression analysis.

The Bayesian specification of a VAR model considers that the parameters in β are random variables that are distributed according to a multivariate Gaussian distribution:

[2.7] $$\beta \sim N(\beta^*, V_\beta)$$

Among the different priors suggested in the technical literature for BVAR models, we have chosen the Litterman/Minnesota prior because it is very well suited for the main use of the model (forecasting) and its structure is grounded on purely statistical considerations, see Litterman (1986).

The Litterman/Minnesotta prior assumes that the underlying model is a random walk plus a drift term:

[2.8] $$(1 - B)z_{i,t} = c_i + u_{it} \quad \forall i$$

Consequently, the mean of the prior on the parameters of the VAR(p) model is given by:

[2.9] $$\beta^* = \begin{cases} c_i = 0 \quad \forall i \\ \phi_{i,j,h} = \begin{pmatrix} 1 & i = j & h = 1 \\ 0 & i \neq j & h \neq 1 \end{pmatrix} \end{cases}$$

The variance-covariance matrix of β is assumed to be diagonal and it is controlled by a vector θ of m hyperparameters. Those hyperparameters describe the properties of this matrix in a parsimonious way:

[2.10] $$diagonal(V_\beta) = \begin{cases} v(c_i) = (\theta_c \sigma_{ii})^2 \quad \forall i \\ v(\phi_{i,j,h}) = (\theta_1 \; F_{i,j} \; d_h \; (\sigma_{ii} / \sigma_{jj}))^2 \quad \forall i, j \quad \forall h \end{cases}$$

The hyperparameter θ_c controls the variance around the prior mean for the intercept. In the same vein, the hyperparameter θ_1 controls the global tightness of the variance for the parameters that describe the dynamics of the model.

The F matrix reflects the analyst's view on the dynamic interaction between the variables of the model. The standard Litterman/Minnesota prior assumes a symmetrical shape:

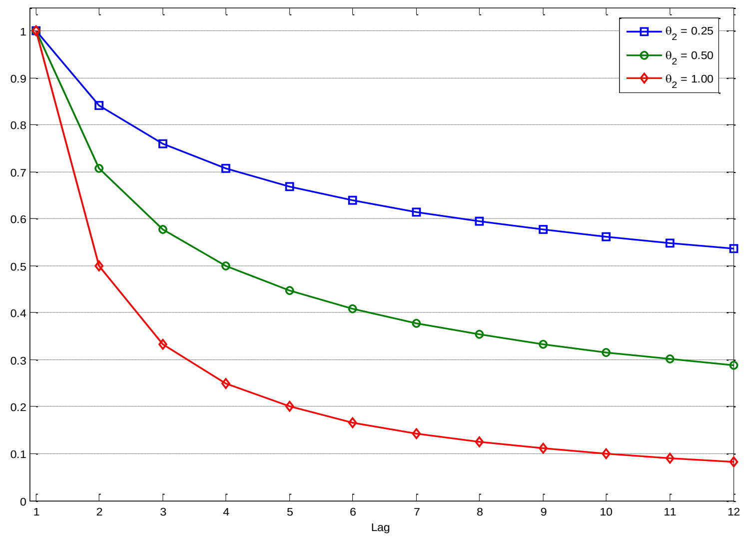
[2.11] $$F_{i,j} = \begin{cases} 1 & i = j \\ f & i \neq j \end{cases} \quad 0 \leq f \leq 1$$

The function d_h quantifies the degree to which the variance is reduced as a function of the lag. These depend on one hyperparameters, say θ_2. The most common representation assumes an harmonic decay:

[2.12] $$d_h = h^{-\theta_2}$$

This function parameterizes the degree of persistence of the model, as can be seen in the following graph:

Figure 1: Litterman/Minnesota prior: harmonic decay function



The complete set of hyperparameters can be placed in the following vector:

[2.13] $$\theta = [\theta_c \quad \theta_1 \quad \theta_2 \quad \theta_3 \quad \mathrm{vec}(F)]$$

Once the prior $\beta \sim N(\beta^{*}, V_{\beta})$ is specified, the estimation is carried out using the Theil-Goldberger mixed method, obtaining:

[2.14] $$\hat{\beta} = ((\Sigma^{-1} \otimes (x'x)) + V_{\beta}^{-1})^{-1}((\Sigma^{-1} \otimes x)'Z + V_{\beta}^{-1}\beta^{*})$$

The corresponding variance-covariance matrix of $\hat{\beta}$ is:

[2.15] $$V_{\hat{\beta}} = ((\Sigma^{-1} \otimes (x'x)) + V_{\beta}^{-1})^{-1}$$

The evaluation of [2.14] and [2.15] requires the prior knowledge about Σ. Usually, this matrix is determined by means of the residual variance-covariance matrix of k univariate AR(p) models or that of an unrestricted VAR(p).

It must be underlined that the estimator in [2.14] is valid under symmetric constraints and as well as under asymmetric ones. In Bayesian parlance, [2.9] and [2.10] define the prior distribution of β and [2.14] and [2.15] the corresponding posterior distribution.

2.2 BVARX Models

Adding exogenous variables to a BVAR has little impact on the formal representation of the model as well as in its calibration and estimation. On the other side, the whole forecasting procedure becomes automatically conditioned on a predetermined path for the exogenous variables.

From a formal point of view, the exogenous variables have a similar role to the intercept, modifying slightly equation [2.1]:

[2.16] $$Z_t = c + \gamma\chi_t + \Phi_1 Z_{t-1} + \Phi_2 Z_{t-2} + \ldots + \Phi_p Z_{t-p} + U_t$$

where γ is a kxm matrix that comprises the impact multiplier of the m exogenous variables χ_t. Calibration and estimation of γ can be made in the same way as c, using a diffuse prior for each of the kxm parameters:

[2.17] $$\gamma_i \sim N(0, \theta_{ij}\,\sigma_{ii}) \qquad \theta_{ij} \to \infty$$

The forecasting procedure is based on the state space representation of the BVARX model and operates feeding it with simulated paths derived from bootstrap resampling from the estimated residuals. In this way, we obtain the complete forecasting density for the endogenous variables conditioned on a preset path for the exogenous variables. From this density we can derive easily point forecasts (e.g. based on the medians), confidence intervals (e.g. based on the corresponding percentiles) and specific paths for the variables than can be plugged in other models to produce confidence intervals, etc.

3 Structure of the model and data issues

In this section we explain the selection of the variables that enter into the model both in the exogenous block as well as in the endogenous block.

3.1 Selection of variables

One of the major drawbacks when building an econometric model that encompasses the main representative magnitudes of an economy is to assess which variables to include. If we look at economic theory, the number of significant variables can be very high, which, in the context of BVAR models described above may result in serious problems of identification and estimation.

Similarly, another element of singular importance is the time span of the time series considered. In this paper we will take data from the first quarter of 1995, which is the time period that is currently available for the current accounting basis of the Quarterly National Accounts published by the INE[1].

Following the scheme proposed by Ballabriga et al. (1998), a suitable selection method, taking into account the constraint that the number of variables cannot be very high given the size of the sample, consists in selecting the basic sectors that have an influence on economic activity and the most relevant associated series. The selection process is represented in figure 2.

Beginning with the private sector, it has to synthesize the decisions of domestic agents in the markets for goods and services, as well the labor market. Regarding the first

[1] At present, the official accounting reference is the 2010 Base. For more information, please see: http://www.ine.es/jaxi/menu.do?type=pcaxis&path=%2Ft35%2Fp009&file=inebase&L=0

point, the level of real activity in the economy will be reflected by production. Obviously, the series of Gross Domestic Product (GDP) in real terms will best reflect the evolution of this variable, because, by definition, is the final result of the production activity of resident producer units.

Also, the inclusion of prices is necessary, being an important point of reference in the decision process related with consumption and investment made by private agents. Prices will be represented by the GDP deflator, which summarizes their behavior from the point of view of supply and demand.

Labor market will be represented by employment, taking as a reference the number of full-time equivalent jobs provided by the National Accounts. Note that the three series finally selected for the private sector are provided by the National Accounts (particularly the Quarterly National Accounts), which has the advantage of providing a consistent and comparable common framework.

When it comes to the public sector, we have summarized its activity by a single variable, in order to maintain the model size within manageable limits. This variable will represent the budget balance (revenues less expenses) associated with the economic cycle. Specifically this variable, so called Net Revenues, will be defined as follows (in brackets shows the number of operation according to ESA 2010):

+ Taxes on production and imports (D.2), mainly comprising: Value added tax (VAT), taxes on imports, excise taxes (tobacco, gasoline, alcohol) and Transfer taxes.

+ Current taxes on income and wealth (D.5), mainly comprising: Income Tax and taxes on profits of corporations.

+ Net social contributions (D.61).

– Unemployment benefits.

The first three components are compiled from the Quarterly Non-Financial Accounts for the Institutional Sectors (QNFAIS), while the latter is provided by the Ministry of Employment and Social Security. Government's net income variable will be expressed as a ratio to nominal GDP.

The foreign sector block shall include the influence of the decisions of economic agents that do not belong to the national economy. Spain's membership of the European Union (EU) has clearly increased the interlinkages between the domestic activity and its main trading partners. Therefore, as a reference of this spillover potential, we will include the GDP volume in the EU.

On the other hand, the impact of trade should be captures not only via quantities but also via price effects, so it seems appropriate to include some measure of competitiveness. The selected series has been the Nominal Effective Exchange Rate against the OECD, which is a multilateral exchange rate weighted by the corresponding indices of domestic prices.

Finally, given the potential role that oil prices may have as a destabilizing factor for the global economic activity, the North Sea Brent price has been included in the model.

As a fourth sector, taking into account its increasing relevance and impact on the economy, we included some measure of the financial sector activity. Firstly, we include interest rates as they represent a determining variable for consumption and investment decisions. More concretely, is the interest rate of new operations provided by credit institutions to non-financial corporations for loans up to one million euros is used to characterize the stance of monetary policy.

Secondly, taking into account potential shortages in the supply of credit and the disruption of traditional transmissions channels of monetary policy, the volume of credit circulation is selected and will be represented by the series of the balance of financing to businesses and households, deflated by core inflation (CPI excluding energy and unprocessed food).

Considering all the selected series as a whole, the distinction between endogenous and exogenous variables becomes evident. The first group will include real GDP, full-time equivalent jobs, GDP deflator, Net Income and financing to businesses and households deflated. Meanwhile, the exogenous variables will be EU GDP, Brent price, nominal effective exchange rate and interest rate of new operations provided by credit institutions to non-financial corporations. This selection also proves useful in the evaluation of the Stability Programme as it mimics the block of external assumptions on which the macro projections are based.

Figure 2. Sectors, variables and observed series

Note: endogenous variables are in blue and exogenous variables are in orange

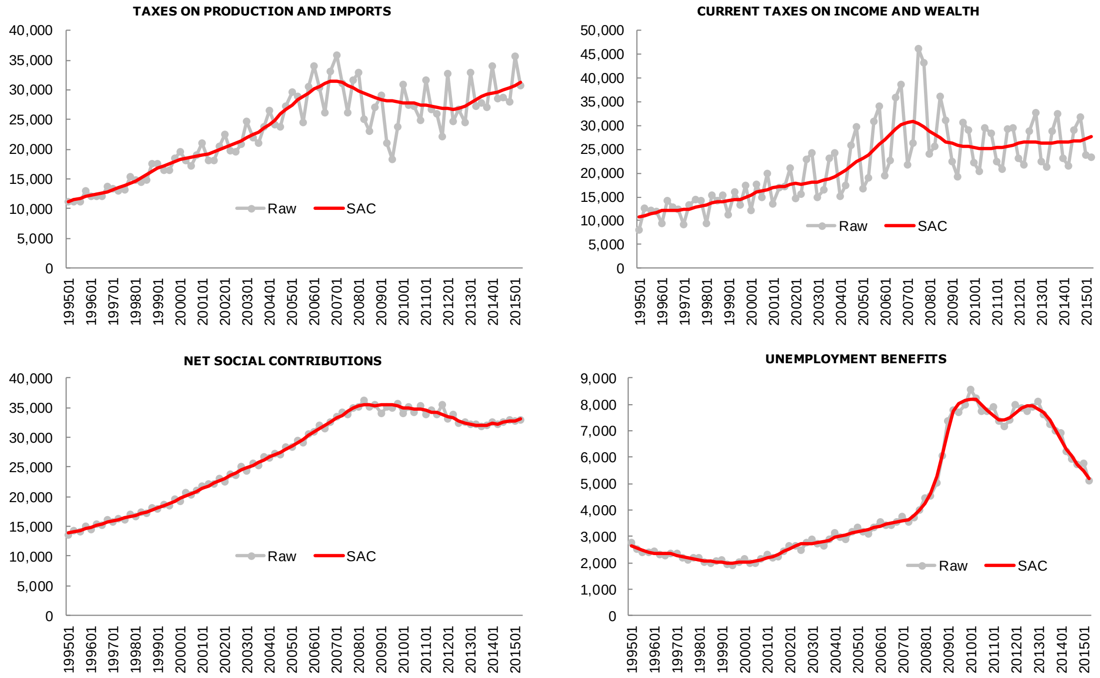
3.2 Preprocessing and transformations

All the variables have to be corrected from seasonal and calendar effects in order to get a signal that is free of possible distortive elements and helps to predict more accurately economic growth.

In the case of the series from the Quarterly National Accounts, they are already published corrected of such effects. For other series, the Tramo-Seats program is used (Maravall and Gómez (1996), Caporello and Maravall (2004)) to correct them, by means of decomposition processes based on univariate models.

The construction of the Government's net income variable deserves special attention. The series of taxes that compose this variable are provided by Non-Financial Quarterly Accounts for Institutional Sectors, specifically by the Public Administration sector account. These variables were corrected from legislative changes by implementing a statistical procedure for outliers correction. This outlier removal significantly improves its correlation with the other variables included in the model. A representation of the raw (uncorrected) series with the corresponding seasonally adjusted series and the series corrected from outliers are shown in figure 3.

Figure 3. Seasonal adjustment and outlier correction of taxes, social contributions and unemploymnet benefits



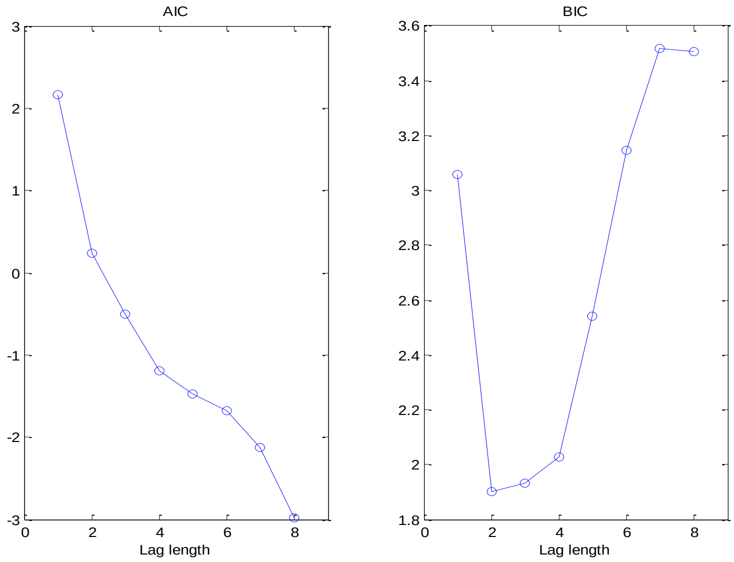
Alternatively, following Ballabriga et al. (1998) we can filter the Government's net income variable using a non-centered four terms moving average. Since the gain function of this filter has zeroes at the seasonal frequencies, we can skip the seasonal adjustment step. In addition, this filtering provides a representation of the data on an annual basis, easing the comparison with the budgetary figures.

Finally, although in VAR models is not strictly necessary, we opted to seek a dynamic relationship between the quasi-stationary series, so we proceeded to take regular differences to the log transformed series, except for the Government's net income variable, that enter in levels (remember that it is expressed as a percentage of GDP), and interest rates, that are included in regular differences to the level.

4 Econometric results

We have used the Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC) to select the lag length p of the BVAR model. As can be seen in figure 4, both criteria suggest quite different values for p. We have chosen the most parsimonious value (the one suggested by the BIC, *p=2)*, as the appropriate lag length of the model.

Figure 4: VAR lag length criteria



We have adapted the Litterman/Minnesota prior to accommodate the fact that the time series included in the model are stationary, as a result of the general application of the difference operator on the log-transformed series. The prior for the mean is now:

[3.1] $$\beta^* = \begin{cases} c_i = 0 & \forall i \\ \phi_{i,j,h} = \begin{cases} \phi_i & i = j \quad h = 1 \\ 0 & i \neq j \quad h \neq 1 \end{cases} \end{cases}$$

The remaining elements of the Litterman/Minnesota remain the same.

The calibration of BVARX model, as opposed to BVAR models, is a relatively new area. The inclusion of purely exogenous variables has a definite impact on the steady state of the model and is at the heart of the forecasting process, rendering calibration based on out-of-sample forecasting performance intractable. As a result, we have relied on a fairly standard and symmetric calibration, e.g. Doan (2010).

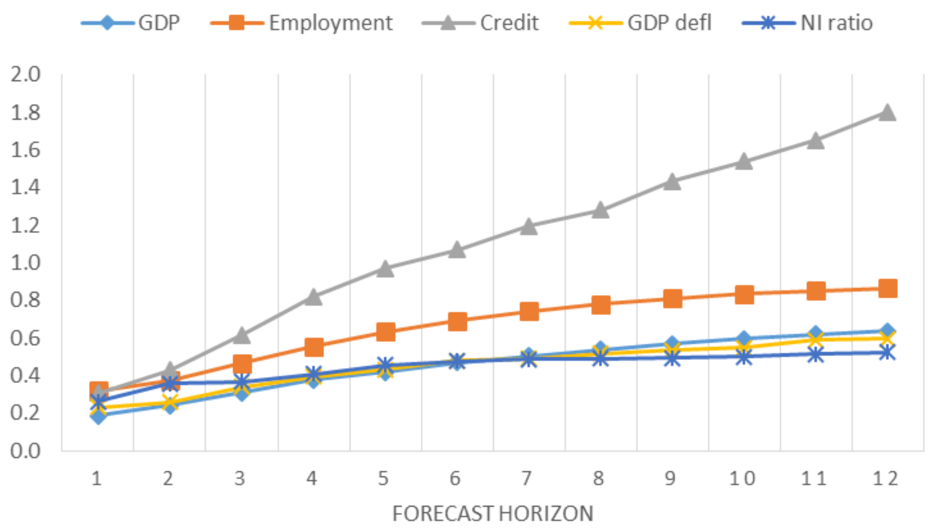
Table 1: BVAR prior calibration

Lag length	2					
			Hyperparameters			
Tightness						
Intercept	10000					
Exogenous variables	10000					
Global	0.20					
Lag decay	0.90					
		Interaction matrix				
	Mean at lag 1	GDP	Employment	Credit	GDP Deflator	Net taxes
GDP	0.75	1.00	0.10	0.10	0.10	0.10
Employment	0.75	0.90	1.00	0.75	0.75	0.75
Credit	0.75	0.90	0.75	1.00	0.75	0.75
GDP Deflator	0.75	0.10	0.10	0.10	1.00	0.10
Net taxes	0.75	0.90	0.75	0.75	0.75	1.00

In order to assess the forecasting performance of the model we have run a backtesting exercise that considers 2005.Q4 as the starting point and projects all the variables at different horizons, from 1 quarter (short-term) to 12 quarters (medium-term).

In all the cases we have computed the mean absolute error (MAE) for all the variables of the model at different horizons. The results are shown in the next graph:

Figure 5: Backtesting of the BVARX model. Mean Absolute Errors (MAE)



The backtesting uses the observed exogenous variables as inputs, leaving aside the thorny problem of forecasting their values on a real-time basis. Of course, this assumption underestimates the size of the forecasting errors, as compared to those that arise from a real forecasting exercise but disentangles the uncertainty that surround the expected evolution of the exogenous variables from the forecasting performance of the model with respect to the core of endogenous variables.

The main points of the backtesting exercise are:

- Although MAE increases with the forecasting horizon, as expected, its growth is quite contained for some variables: GDP, inflation and net income.
- The forecasting performance of the model for employment is quite good for short-term forecasts (up to 4 quarters) but it deteriorates somewhat for longer forecasting horizons.
- Forecasting credit during the out-of-sample period (2004-2015) appears as a daunting task. This result may be due to the financial conditions observed during the last part of the period (2008-2015) that departs from the historical pre-crisis pattern.

The Bayesian component of the BVAR/BVARX models provides an easy and flexible way to introduce non-sample information, by means of alternative prior distributions or alternative calibration of a given prior, see Karlsson (2012). In order to assess the role of the prior information, we have repeated the backtest with the following alternative calibrations of the Litterman/Minnesota prior:

- BVARX: The baseline calibration described in table 1.
- BARX: The univariate equivalent of the BVARX model, without interactions among the variables: F=I.
- VARX: The unrestricted VAR model, in which no prior information is considered: F=I, $\Theta_1=\infty$, $\Theta_2=0$.
- MIN: The standard Litterman/Minnesota prior: f=0.5, $\Theta_1=0.2$, $\Theta_2=1$.
- Fmax: The BVARX model with no constraint on the interactions: f=1, $\Theta_1=0.2$, $\Theta_2=0.9$.

Table 2: Backtest using alternative calibrations

GDP	BVARX	BARX	VARX	MIN	BVARX F max
1 Quarter	**0.19**	0.19	0.25	0.21	0.21
1 Year	**1.11**	1.15	1.45	1.19	1.26
2 Years	**3.04**	3.11	3.95	3.24	3.30
3 Years	**5.46**	5.52	7.32	5.91	5.87
Employment	BVARX	BARX	VARX	MIN	BVARX F max
1 Quarter	**0.32**	0.33	0.36	0.33	0.34
1 Year	1.71	1.73	2.08	1.72	**1.68**
2 Years	4.56	4.48	5.88	4.61	**4.47**
3 Years	7.91	**7.62**	11.11	8.27	8.04
Credit	BVARX	BARX	VARX	MIN	BVARX F max
1 Quarter	0.31	0.32	0.35	**0.30**	0.31
1 Year	**2.17**	2.48	2.77	2.18	2.25
2 Years	**6.68**	7.57	9.96	6.75	7.04
3 Years	**13.10**	14.64	22.26	13.29	14.02
GDP defl.	BVARX	BARX	VARX	MIN	BVARX F max
1 Quarter	0.23	0.23	0.24	**0.22**	0.22
1 Year	1.22	1.26	1.17	1.16	**1.13**
2 Years	3.14	3.27	2.91	2.94	**2.83**
3 Years	5.41	5.65	5.03	5.13	**4.96**
NI ratio	BVARX	BARX	VARX	MIN	BVARX F max
1 Quarter	0.26	**0.25**	0.31	0.26	0.26
1 Year	1.39	**1.24**	1.67	1.36	1.44
2 Years	3.30	**2.84**	4.18	3.25	3.47
3 Years	5.33	**4.51**	7.03	5.28	5.66

Note: accumulated end-of-period errors. Red bold figures indicate best results.

Analyzing the results by variables:

- The best results for GDP and credit are obtained using the BVARX model.
- The BVAR with unrestricted interactions (Fmax) provides the best results for inflation, especially for medium-term forecasts.
- The best forecasts for net income are obtained by means of the univariate model. This result may be due to its very persistent nature that collides with the more volatile dynamics that characterizes the remaining variables.

- All the models yield a similar performance for employment, providing the baseline calibration good results.

Two issues arise when comparing the different models:

- The VARX specification provides the worst results for all the variables irrespective of the forecasting horizon, providing a decisive argument for the use of Bayesian elements.

- Corroborating the previous point, the BARX provides acceptable results, confirming the convenience of introducing the type of prior information related to the dynamics embedded in the BVARX model.

The model generates forecasts taking as given a scenario for the exogenous variables and assuming as initial conditions the forecasts provided by AIReF's real-time forecasting model (Cuevas et al., 2015). In this way, we combine a permanent link to the most updated short-term forecasts with a medium-term view on the exogenous variables.

The first step in the forecasting procedure is the definition of a scenario for the exogenous variables of the model (the X in BVARX). For ease of implementation we consider this scenario in annual terms and transform it to the quarterly frequency using the relationships between rates of growth at both frequencies, see Quilis (2015b). The next table represents a likely scenario:

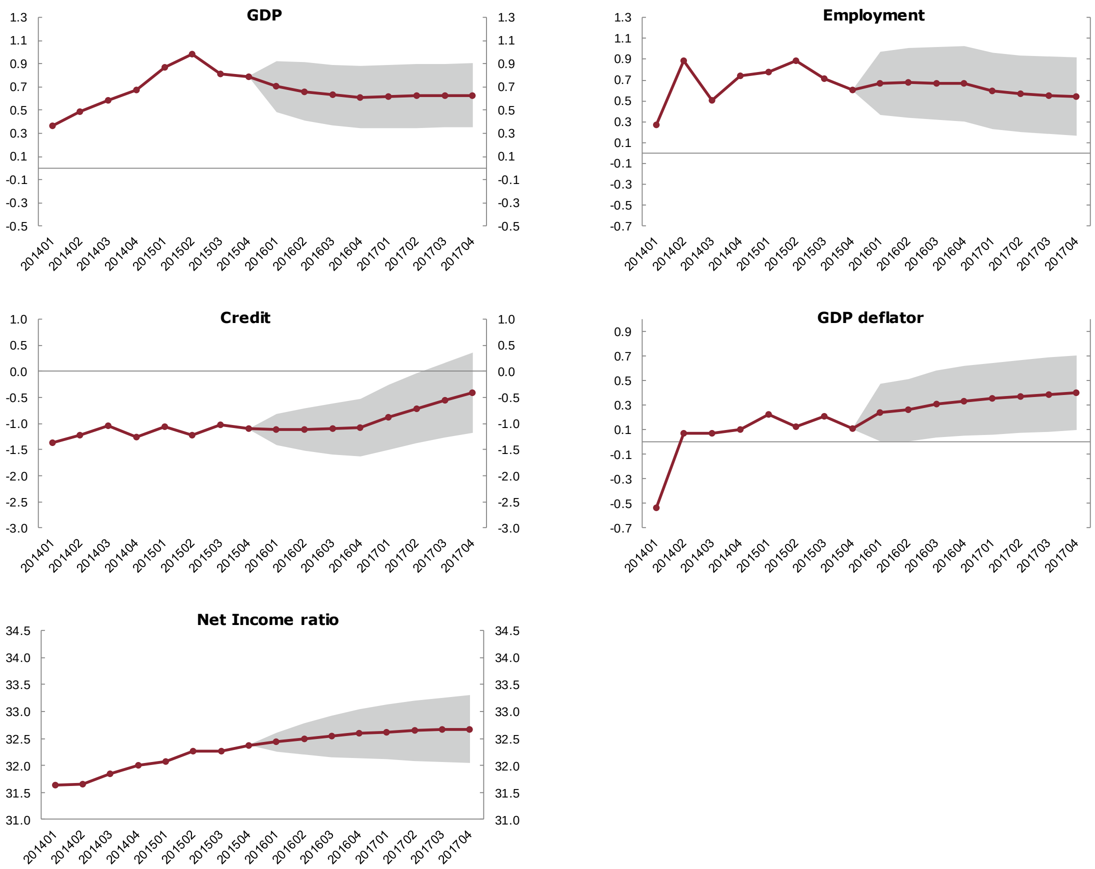
Table 3: Scenario for the exogenous variables (2016-2017)

Year	Interest rates	EU GDP	Euro nominal effective FX	Oil price (Brent)
	Level at end of period	Annual rate of growth	Annual rate of growth	Average level
2016	3.3	1.8	1.2	36.1
2017	3.1	1.9	-0.2	42.2

The second step uses the most updated forecasts for the endogenous variables as if they were observed data and computes the joint forecasting density for the endogenous variables conditioned on the scenario for the exogenous variables, as explained in section 2.

With the corresponding real-time forecasts for 2016.Q1-2016.Q2 and the scenario depicted in table 3 we compute the forecasts for the endogenous variables and their corresponding confidence intervals for the first eight periods. The results are show in figure 6.

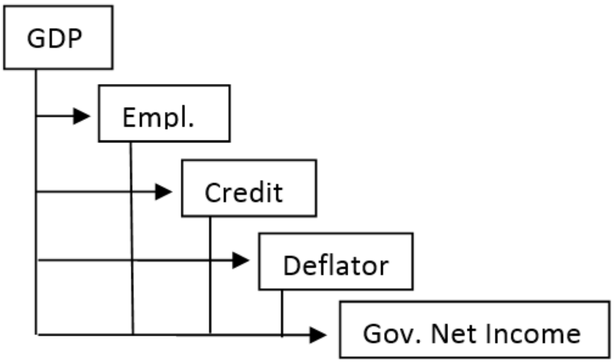
Figure 6: BVARX forecasts



Although forecasting is the main use of the BVARX model it can also be used for structural analysis. Of course, structural analysis requires the identification of the “deep” or orthogonal shocks that buffet the system. This identification process can adopt many forms and transforms the initial reduced form VAR model in a structural one (SVAR), see Lütkepohl (1991) and Stock and Watson (2008) for an in-depth exposition of SVAR models.

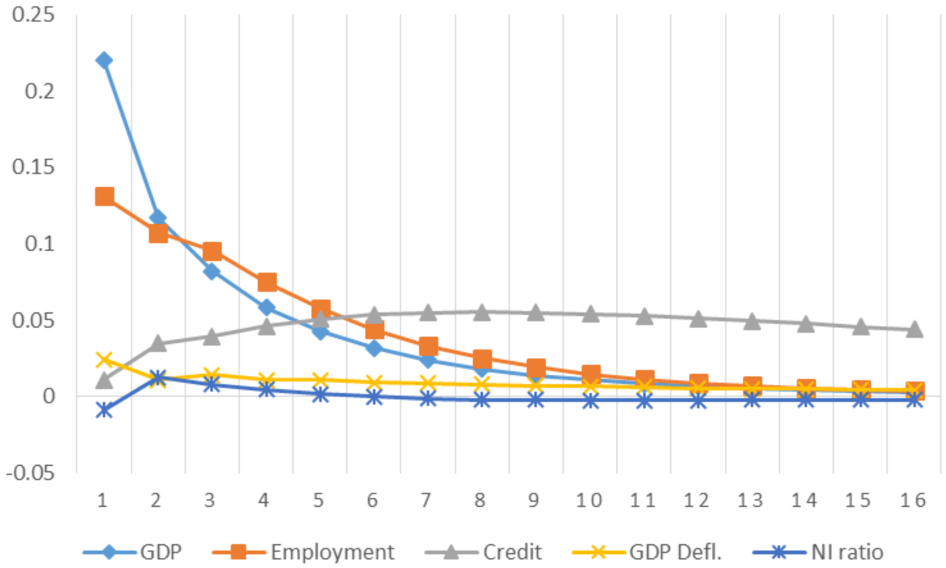
Assuming the recursive identification (Cholesky orthogonalization) depicted in figure 7, we can estimate the impulse response functions implied by the SVAR model.

Figure 7: Recursive identification of the BVARX model



As an illustration, the next graph shows the response function of the variables of the model to a one-sigma shock to GDP.

Figure 8: Response functions to a 1 s.d. GDP shock



5 Conclusions

BVARX models provide a convenient way to generate conditional forecasts for the key macroeconomic variables of the Spanish economy, providing at the same time an objective way to assess the consistency of macroeconomic scenarios and the likelihood of alternative projected paths for the endogenous variables.

There are several promising lines of research that may widen the scope of the BVARX model. Enlarging the model to accommodate mixed frequencies (quarterly and monthly) will allow us to include the information provided by the dynamic factor models, extending in this way their usual forecasting horizons from 1-2 quarters to 4-6 quarters.

The combination of BVARX models with Vector Error Correction models (VEC) is also a promising avenue. Since VEC models incorporate explicitly low-frequency information (via cointegrating relationships), they complement naturally BVARX models in first-differences. A possible way to combine both models would be by means of a prior related to the steady-state of the BVARX model. This prior on the steady-state can include information provided by the VEC model, see Villani (2009) for an in-depth analysis of these priors.

Another line of research is linked to the use of large-scale optimization techniques (e.g. genetic algorithms or simulated annealing) to calibrate the hyperparameters of the BVARX model, adapting existing procedures designed for BVAR models, see Quilis (2015c).

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