Working Paper

WP/2014/1

Understanding financial sustainability

Abstract

This note develops a stylized "real business cycle" model of an open economy, in order to analyze the dynamics of deficit and debt ratios to GDP and the sustainability of public finances. The optimal decisions of individuals are a function of real variables only and there is no unemployment or frictions. However, the level of prices (exogenously given) matters because the value of pre-existing debt (held by non-residents) is fixed in nominal terms, and because fiscal policy makers take decisions in nominal terms, taxing nominal income, giving nominal transfers, and spending in goods and services at current prices. Fiscal policy follows a rule to stabilize the debt ratio at a certain target level and financial markets react to changes in the debt ratio following a rule of thumb. This setting mimics the environment of an economy in a monetary union in which there are free flows of goods and capital, prices are given and interest rates are determined by the monetary rate set by the common central bank plus a risk premium that depends on the assessment of financial markets participants of the sustainability of public finances. The interaction of fiscal policy makers, consumers and financial markets participants determine the time path of the economy, the sustainability of public finances and the probability of debt default.

Written by José Marín Arcas
Revised by Carlos Cuerpo Caballero y Miguel Ángel García Díaz

Contacto AIReF: C/José Abascal, 2, 2º planta. 28003 Madrid.Tel. +34 917 017 990
Email: Info@airef.es. Web: www.airef.es

Executive summary

This note develops a "real business cycle" model, in which economic agents react to real variables only and there is no unemployment or frictions. Although nominal variables would not affect the equilibrium of the economy when fully indexed, exogenous price shocks have real effects because they change the value of nominal variables like the stock of debt in real terms, inducing further reactions of fiscal policy makers and financial markets. The formal representation of this economy has been kept very simple. **Individuals live an uncertain number of years. They have the same preferences about consumption and leisure, but different endowments of labour capabilities**. Labour endowments are unequally distributed between them and change for each individual from year to year, because of sickness, or invalidity, or any other unpredictable event. Although individuals are uncertain about their future ability to work, or even if they will be alive, they do not save. Instead, there exists a collective insurance system run by the government as a mechanism of taxes and transfers with redistributive effects (between individuals in each year) and stabilizing effects (between prosperity and adversity situations of the same individual in different years). Such mechanism takes the form of **a negative linear income tax**, which guarantees the transfer of a minimum income to each individual in each period and taxes income at a flat rate. Apart from this expenditure in transfers, the government provides goods and services affecting the output of the economy. Section 1 analyzes the existence of period by period equilibria and the steady state of this economy in a deterministic setting.

The dynamic path, the stability of the economy, and the sustainability of public finances are considered in Section 2. **Sustainability is understood in this paper as the ability of policy makers to follow certain rules that preserve the stability of the economy around a steady state equilibrium**. There are many possible rules to enhance fiscal rectitude. This paper opts for a general and flexible type: the (u,v) rule. The (u,v) rule adjusts the primary balance ratio taking into account the distance between the current level of the debt and its equilibrium or target value, and the gap between the current and the equilibrium value of the primary balance. This section deals first with the **dynamic effects of active and passive fiscal policies** (multipliers and automatic stabilizers, respectively) under this rule in a deterministic setting, previous to the consideration of interest rates

reactions in financial markets to changes in the debt ratio. Finally, section 2 deals with a stochastic setting in which **the economy is subject to population, technology, preferences and purely nominal random shocks**. This set-up provides a "realistic" environment for cyclical fluctuations in which the interaction of the real economy, fiscal policies and financial markets generates the dynamic path of all variables. By simulating the behavior of the economy a sufficiently large number of times, it is possible to estimate the probability of default on government debt and the ensuing collapse of the economy under different initial assumptions.

The **fiscal rules prevailing in the Spanish economy** are examined in Section 3. These rules, like those enshrined in the EU Treaties, are in substance balanced budget rules, with a number of escape clauses which might be invoked under exceptional circumstances. Although they are very complex in detail, a succinct description and formalization in the context of the described model is sufficient to understand their main thrust and role in preserving the sustainability of public finances. Fiscal discipline is stricter in Spanish regulations than requested by the EU fiscal framework. The period of transition until the objectives of budgetary stability (recording surpluses) and financial sustainability (a debt ratio below 60% of GDP) are attained was regulated in a particularly stringent manner in the Spanish Organic Law of Budgetary Discipline and Financial sustainability of 2012. Transitory provision 1 of this law sets a deadline for the attainment of the targets in 2020, at the latest. In addition, the law requires, among other conditions, that non-financial spending of all levels of government does not grow more than real GDP until the debt ratio is below the reference value of 60% of GDP.

Notwithstanding the necessary simplifications made by the stylized model, several important takeaways can be drawn from the analysis presented in this paper, as summarized in the final section. First, any announced policy "measure" should precisely specify its future fiscal implications, allowing for a fully-fledged assessment; a trivial conclusion although frequently ignored by policy makers. Second, the arithmetic calculations for the possible paths of the debt and deficit ratios in the Spanish economy under different scenarios, together with the projected increase of the debt ratio by the Spanish Government up to close to 100% of GDP in 2017, inevitably calls for **an extension of the 2020 deadline for reaching the 60% of GDP target**, currently present in the Spanish law. Furthermore, the necessary extension of the deadline makes more unpalatable the requirement limiting the growth of non-financial government spending.

This requirement could be circumscribed to the periods in which there is no **overall surplus in government accounts, a provision which would still keep the debt ratio on a reasonably steep path of decline**.

Content

1 A Stylized "real business cycle" model

This note develops a stylized "real business cycle" model in the spirit of the seminal work by Kydland and Prescott (1982) in order to analyze the dynamics of deficit and debt ratios to GDP and the sustainability of public finances.

Individuals live an uncertain number of years in this economy. They can die next period with a certain probability or may be sick and partially, or totally, unable to work. We assume for simplicity that consumers do not discount the future and they are not individually trustworthy enough to borrow. Instead of saving to cater for the future, there is a pooling of risks through a social insurance scheme run by the government, which provides a minimum guaranteed income in each period. Consumers have identical preferences and maximize a period by period utility function of consumption and leisure, paying taxes on their income at a given rate. The government collects income taxes, pays the minimum income to everybody, provides productive services to the economy and borrows or lends abroad at a given interest rate to finance its budget imbalances.

Fiscal policy affects the levels of output, consumption and leisure and creates an incentive for private agents to demand lax fiscal policies that would increase their current utility at the expense of the future. In an open economy, agents would be in favor of ever larger budget deficits, as long as the imbalance can be financed by non-residents and the debt repaid by future individuals. The inter-temporal budget constraint of the government imposes, however, the restriction that this policy has to be sustainable and perceived as such by financial markets participants (non-residents) for them to hold public debt.

Sustainability is preserved if the debt ratio to GDP is stable or declining. The model used in this note illustrates that reducing the debt ratio to a very low level is desirable not only to safeguard economic stability, but also for reasons of efficiency and intergenerational equity. First, the "burden of the debt" is shifted to the future, reducing consumption possibilities as it would imply higher tax rates. The convexity of the welfare losses associated with the distortionary effects of taxation imply that utility gains today will be surpassed by losses in the future (see Barro 1979 for the original argument on the efficiency of tax smoothing). Second, raising the debt ratio to expand current consumption shifts part of the burden of the debt to future (yet unborn) individuals, which will benefit less from the government and suffer from a lower capital stock (see Diamond 1965 and Musgrave 1986).[1]

1.1 Description of the economy and optimal decisions of individuals

All individuals have identical preferences and live an uncertain number of periods. In each period individuals (there are ***n*** of them) have a given endowment of labour (some of them more and some less than the average, which is equal to one), and they have to decide how to allocate their time in this period between leisure and work. Aggregate leisure is ***l*** and aggregate work is (***n-l***). They maximize a period by period Cobb-Douglas utility function[2]:

$$U(c,l) = c^{\alpha} \cdot l^{1-\alpha}, \quad with \ \ \alpha \in (0,1) \qquad [1]$$

where ***c*** is consumption of the single good produced in the economy. Capital letters will refer to variables at current prices in period t. Unless otherwise specified, all variables will be dated at time t, and the sub-index will be dropped. Disposable income is equal to gross income (***Y***, the output of the economy) after paying tax (***T***):

$$T = \tau \cdot Y \qquad [2]$$

where τ is the flat tax rate. As agents have different initial endowments of labour skills in each period, the government follows a redistributive policy to guarantee a minimum income. In this way, taxes net of transfers are determined as a progressive linear income tax (equal to: $-I + \tau \cdot Y$), where ***I*** is the minimum guaranteed income, which has distortionary effects on the consumption-leisure choice.

The available technology is represented by the aggregate production function:

---

1 For an application of these criteria to the Dutch case, see van Ewijk *et al*. 2006.

2 As long as individuals with homothetic preferences only differ in their initial endowment of labour, their decentralized decisions will result in the same aggregate outcome as the decision of a “representative” agent

$$y = k \cdot (n - l) + \frac{G}{p} \qquad [3]$$

where ***k*** is the productivity of labour, ***G*** is aggregate public spending in goods and services (government consumption and investment) and ***p*** is the price of the single good in the economy. Capital is absent. This representation of production possibilities has been chosen for reasons of simplicity[3]. The aggregate production function is the sum of individual production functions of individuals with identical productivity and using public services equally:

$$y_i = k \cdot (n_i - l_i) + \frac{G}{n \cdot p} \qquad [4]$$

Prices are exogenous, growing at rate π:

$$p_{t+1} = (1 + \pi_t) \cdot p_t,$$

We assume that population and labour productivity grow at constant trend rates ***η*** and ***ξ***, respectively, although their actual rates of growth might be subject to random shocks. The rate of interest (***r***) is bigger than the nominal rate of growth (***g***) of the economy by a risk premium:

$$(1 + r) > (1 + g) = (1 + \eta) \cdot (1 + \xi) \cdot (1 + \pi) = (1 + \gamma) \cdot (1 + \pi)$$

Where ***γ*** is the rate of growth of real output. In each period t, individuals have as resources income net of taxes (Y-T) and income transfers or pensions (I; i.e. I/n per person) which they use to consume (C=p·c). The aggregate budget constraint for consumers is:

$$C = p \cdot c = Y - T + I = (1 - \tau) \cdot [p \cdot k \cdot (n - l) + G] + I \qquad [5]$$

[3] Public spending G in goods and services is "productive" in the sense that it represents public spending in education, health, justice, infrastructure services provided by public capital, etc. Government Non-financial spending is thus either "productive" (represented by G) or "redistributive" (represented by I).

Maximizing the utility function subject to this single budget constraint, the solution of the optimization problem is:

$$c^* = \alpha \cdot (1-\tau) \cdot \left[k \cdot n + \frac{G}{p} + \frac{I}{p \cdot (1-\tau)} \right]$$ [6]

$$l^* = (1-\alpha) \cdot [n + \frac{G}{p \cdot k} + \frac{I}{p \cdot k \cdot (1-\tau)}]$$ [7]

Replacing the value of leisure from [7] in [3], the equilibrium output is:

$$y^* = \alpha \cdot \left(k \cdot n + \frac{G}{p}\right) - \frac{1-\alpha}{1-\tau} \cdot \frac{I}{p}$$ [8]

Note that, ceteris paribus, consumption and leisure are increasing functions of the minimum guaranteed income. Consumption is also a decreasing function of the tax rate, while leisure is increasing in the tax rate. Output is an increasing function of *G* and a decreasing function of τ and *I*. Fiscal policy has real effects. Lower taxes and/or higher public spending in period t increase the utility of the individuals living in t[4].

1.2 Period by period equilibrium

The **structure of the economy** is determined by a specification of the exogenous variables and parameters $\{n_0, k_0, \eta, \xi, \alpha, r, p_0, \pi, B_0\}$. As explained before, the government collects taxes, pays transfers and interest on public debt (***B***, which takes the form of one-year maturity bonds yielding nominal interest rate r), and spends on goods and services (***G***), subject to a flow budget constraint:

$$B_t = B_{t-1} - T_t + I_t + G_t + r \cdot B_{t-1}$$ [9]

Fiscal policy is a set of rules to determine the time paths of the variables $\{I, G, \tau\}$ controlled by the government in order to attain an announced target for the debt ratio or

[4] For those interested in checking, some mathematical notes are provided in Annex 1. For each individual (i=1,…,n), consumption, leisure and output are given by the same formulas [6]-[8], just replacing n by n_i, G by G/n, and I by I/n. Notice that absolute inequality in income and leisure between individuals i and j is not affected by fiscal variables, but absolute inequality in disposable income and consumption is lower the higher the tax rate.

the primary balance ratio. Public finance developments are determined by the reaction of the individuals in the economy (and later on also of financial markets participants) to fiscal policy.

The equilibrium of the economy in each period requires that supply and demand of goods are equal for a given specification of {*I,G,τ*} and individuals adopt optimal decisions on consumption and leisure:

$$Y_t = C_t + G_t + X_t - M_t \qquad [10]$$

Where ***X*** are exports and ***M*** imports. Note that consumers do not save, and the external balance (***EB***) is equal to the budget balance (***D***). The following chain of accounting identities shows that in this economy the primary balance (***S***) equals net exports, while the income balance corresponds to the interest payments on government debt:

$$EB_t \equiv Y_t - C_t - G_t - r \cdot B_{t-1} \equiv X_t - M_t - r \cdot B_{t-1} \equiv Net\ exports + Income\ balance$$

$$EB_t \equiv D_t \equiv T_t - G_t - I_t - r \cdot B_{t-1} \equiv S_t - r \cdot B_{t-1} \equiv Primary\ balance - Interest\ payments$$

The existence of equilibrium in each period is guaranteed by the assumption that there are perfect international markets to import or export consumption goods at the current price, and capital markets to lend and borrow at the current interest rate. Such assumption is necessary to allow next exports to play an adjustment role in balancing aggregate supply and demand, and capital flows to finance this adjustment through the government budget balance.

1.3 Steady state equilibrium

Consider first the case of an economy growing without fluctuations or random shocks. A **steady state equilibrium** in this economy is characterized by two conditions:

1. **Supply and demand equilibrium in each period** at the constant rate of nominal output growth $g = (1+\eta)\cdot(1+\xi)\cdot(1+\pi)-1$, and

2. **Stationary public finances**, defined by a constant tax rate and constant ratios to output of government debt (the target of fiscal policy) and deficit, public expenditure in goods and services and transfers.

In the steady state, by 2, the debt has to grow at the rate g, which implies:

$$B_t = B_{t-1}(1+g) \; and \; B_t = B_{t-1} - D_t \rightarrow \; D_t = -g \cdot B_{t-1} \rightarrow$$

$$D_t = \tau_t \cdot Y_t - I_t - G_t - r \cdot B_{t-1} = \; -g \cdot B_{t-1} \rightarrow S_t = (r-g) \cdot B_{t-1}$$

Hence, assuming r>g and B_{t-1}>0, the primary balance in the steady state has to be in surplus: the economy has to generate a trade surplus in order to finance the "**burden of the debt**" given by (r-g)B_{t-1}. Notice that it might happen for some stretch of time that r<g, which implies that there is no real burden in contracting debt because the growth rate of output is bigger than the financial cost of the debt. This situation can stimulate current consumption and the accumulation of debt for a number of periods, leading to a higher burden on future consumers if the interest rate, as it is to be expected, rises again above the nominal growth of the economy.

From the last expression for the budget balance in the steady state we can express the value of G as a function of the other variables in the budget constraint:

$$G_t = \tau_t \cdot Y_t - I_t - (r-g) \cdot B_{t-1}$$

Given the values of B_{t-1}, τ and I, plus the other parameters defining the structure of the economy, the value of G that stabilizes the initial debt ratio can be calculated by inserting in the last equation the equilibrium value of output, from [8], in nominal terms:

$$G^* = \frac{1}{1-\tau\cdot\alpha}\cdot\{[\tau\cdot\alpha\cdot p\cdot k\cdot n]-(r-g)\cdot B_{t-1}\}-\frac{I}{1-\tau} \qquad [11]$$

It can be checked that: $\frac{\partial G}{\partial B_{t-1}} < 0, for \; (r>g), \; \frac{\partial G}{\partial I} < 0$. Notice that the value of G* would become negative when τ tends to 0 and when τ tends to one, so that there is a tax rate which maximizes the revenue collected in the equilibrium steady state to be used to finance government spending.

Finally, by replacing G* from [11] into [8], the steady state equilibrium of output is:

$$y^* = \frac{\alpha}{1-\tau\alpha} \cdot \left[kn - \frac{(r-g)B_{t-1}}{p}\right] - \frac{I}{(1-\tau)p} \quad [12]$$

which is positive for a zero tax rate and negative when the tax rate approaches 1. The spreadsheet accompanying this document, named *Equilibrium*, shows for a given level of spending in goods and services, or alternatively, for a given level of transfers, how the choice of consumers changes with the tax rate when there is no debt and the budget is balanced.[5]

1.4 System dynamics and sustainability of public finances

1.4.1 The concept of sustainability

Public finances are sustainable when the steady state equilibrium is stable under a given policy rule, i.e. when the application of a rule steers the economy from the starting position towards a steady state equilibrium. The steady state equilibrium represented in equation [12] is unstable as such. Given the initial level of debt, any small change in one of the three variables $\{I, G, \tau\}$ defining fiscal policy in that equilibrium, keeping the other two constant, would make public finances non-stationary, with the debt ratio increasing or decreasing without limit. Similarly, a change in any of the variables defining the structure of the economy or the exogenous variables would also make public finances non-stationary for a constant policy. The analysis of the stability of the steady state equilibrium requires thus the specification of a rule governing the adjustment of fiscal policy variables when they are not in steady state equilibrium.

The best known rule of fiscal discipline is to balance the budget in every period. A balanced budget rule entails a gradual decline towards zero of the debt ratio, which is not necessarily the optimal value.[6] A different, more flexible type of rule, which aims at

[5] Available for download in our website: www.airef.es

[6] The seminal contribution in Reinhart and Rogoff (2010) created controversy and spurred research on the existence of a concave relationship between public debt and growth, with discussions on possible thresholds for debt-to-GDP ratios from which debt becomes

making the debt ratio converge to any predetermined target respecting the operation of automatic stabilizers, the (u,v) rule, was analyzed in Marín (2002). This rule can replicate the effect of a balanced budget rule by setting the target for the steady state debt ratio equal to zero, but can also be used to explore the consequences of choosing a different target, like stabilizing the debt ratio at 60% or 100%.

1.4.2 The (u,v) rule

The dynamics of the debt level is given by the equation

$$B_{t+1} = (1 + r_t) \cdot B_t - S_t \quad [13]$$

where r is the nominal rate of interest and S is the primary balance. The dynamics of the debt-to-GDP ratio (b=B/Y) is given by the equation:

$$b_{t+1} = \frac{1+r_t}{1+g_t} \cdot b_t - s_{t+1} \quad [14]$$

where s=S/Y is the primary balance-to-GDP ratio. Together with this flow budget constraint, we consider a general rule that adjusts the primary balance ratio taking into account the distance between the current level of the debt ratio (b_t) and its equilibrium value or objective of convergence (b*), and the gap between the current (s_t) and the equilibrium value of the primary balance $s^* = \frac{r_t - g_t}{1+g_t} \cdot b^*$:

$$s_{t+1} - s_t = u \cdot (b_t - b^*) - v \cdot (s_t - s^*) \quad [15]$$

The conditions for global stability of the system are $v > \frac{r-g}{1+r}$ and $u > v \cdot \frac{r-g}{1+g}$.[7]

1.5 Comparative statics of fiscal policies around the steady state

Before considering the introduction of random shocks (to population, preferences, technology etc.), and before taking into account possible reactions from the financial

---

detrimental for long-term growth. See Herndon et al. (2014) for a rebuttal of Reinhart and Rogoff results.

[7] Those interested in the formal analysis of stability conditions in the dynamic system of differential equations defined by equations [14] and [15] in the case of constant interest and growth rates will find in Annex 1 some mathematical notes.

markets, it is worth exploring how fiscal policies operate in a static, deterministic world, around a steady state equilibrium in which prices and interest rates are constant. This section focuses on active fiscal effects. Active effects, i.e. fiscal multipliers, result from discretionary decisions of policymakers while passive effects (analyzed in the next section) , i.e. automatic stabilizers, result from the response of the existing structure of revenue and expenditure to an exogenous change, like a shock to technology or preferences, when existing regulations are applied. In both cases, to keep public finances sustainable, fiscal authorities have to specify how to implement the rule of fiscal discipline after the initial change.

Discretionary fiscal policy in this model consists in selecting a combination of three of the four variables at hand for fiscal authorities: the tax rate, the ratios to GDP of government spending in redistributive transfers and in goods and services, and the target debt ratio. Having set up three of these variables, the fourth is a residual which has to be adjusted to satisfy the budget constraint of the government and to preserve the sustainability of public finances. In the implementation of the sustainability rule, it is possible to use as instruments the tax rate and the two expenditure ratios. Discretionary measures of fiscal policy can affect the levels of real variables in the long run, but not the trend rate of growth of real output, which is determined by the constant trend rates of population and productivity growth. Any discretionary change in fiscal policy requires, in order to be well-defined, the specification of the new steady state or target levels for the fiscal variables, as well as the instruments used in the adjustment process. It is possible then, by comparing the paths followed by all relevant variables, to differentiate between the short-term impact of fiscal multipliers when the policy change is introduced, and their steady state values once the new equilibrium is attained. It is also possible to measure the results in terms of the evaluation that individuals themselves would make of the changes with respect to the baseline, according to their utility function.

In the analysis presented in this section and the next, the starting point for the parameters of the economy and the exogenous variables are:

$$\{n_0 = 1, k_0 = 1, \eta = 0.005, \xi = 0.015, \alpha = 0.8, \ r = 0.045, \ p_0 = 1, \pi = 0.02, B_0 = 0.83\}$$

The fiscal variables are: {τ=0.395;I/Y=20.95%;G/Y=17.11}. The parameters for the implementation of the fiscal rule are (u,v)=(0.05;0.5). All these values have been chosen to replicate approximately some stylized features of the Spanish economy, as we will see later. The qualitative conclusions of the analysis are robust to changes in this specification of values. The results of the different exercises can be checked using the spreadsheet *Comparative Statics* accompanying this document.

In order to limit the variety of possible exercises of comparative statics, α is kept constant at 0.8 as well as prices, interest rates, productivity, and population growth. We assume that the policy instruments are used one at a time, both when taking a discretionary measure and when implementing the rule, instead of combining them. We explore successively the consequences of initial discretionary changes in one direction, both temporary and permanent, in the tax rate, in transfers and in productive public spending, as well as changes in the long term target for the debt ratio, when the adjustment variables are each one of the three possible policy instruments. The effects of changes in the opposite direction are symmetric. Tedious as these exercises may be, they are the building blocks to understand the dynamic path of the economy when it is subject to random shocks.

1.5.1 Changes in the tax rate

Starting with a tax rate of 39.5%, which initially also used as the instrument of control to implement the fiscal rule, and a debt ratio of 100%, which is intended to be stabilized at that level, a shock of 1 percentage point in the tax rate is simulated. As could be expected, agents react by lowering their labour supply. Output is also negatively affected by 10 basis points (bp, in what follows), implying a short-run multiplier of -0.1. The deficit and the debt ratio also fall initially, 1 and 0.8 percentage points (pp) respectively. After the first year, the tax rate starts falling gradually because it has to be adjusted downwards in order to steer the debt ratio back to 100%, and all the initial changes unravel in the course of a few years. Nevertheless, the debt ratio continues falling to a minimum of 98.9% three years later because the tax rate falls only little by little, and the subsequent rebound is very slow, so that interest payments are also kept below the long term level for a number of years.

What happens if, instead of reversing the change in the tax rate, public spending is raised to bring the debt ratio again to 100%? In this case, the size of the government is expanded and there are permanent effects on the economy. If the expansion takes the form of higher transfers, individuals react by working and consuming less, so that output and consumption fall in the long term and utility also declines. Because the sequence of change is first to raise taxes and then increase transfers, output growth, as well as the debt and deficit ratios, converge again to their initial values from below. The tax rate remains 1pp above the starting point, but the ratio of transfers to GDP can only increase by 0.9pp due to the lower level of steady state output.

In the case in which the additional tax burden is used to raise government spending in goods and services, the long term consequences are different from above. Although individuals react again adjusting consumption and labour supply, the increase in productive public spending raises output growth, so that the long term level of output is higher than in the baseline, as well as the ratio of productive government spending. The utility individuals end up with, however, is not significantly different from the situation in which transfers were increased in the previous case. In this case, the transitory impulse on output growth persists for six periods and the debt ratio declines up to 3pp in the fifth year, before rising again slowly to the steady state level.

1.5.2 Changes in transfers

A discretionary change of 1pp of nominal GDP in transfers has only temporary effects when the stabilization of the debt ratio is pursued by further adjusting transfers in future periods. The initial impact of higher transfers is negative on labour supply and output growth (with a short-run multiplier of -0.3), but positive on consumption and utility. The higher consumption is financed through a higher government (equal to the external) deficit. The debt ratio tends to increase (2pp in two years), and only as transfers are adjusted downwards the growth rate rises again above the trend rate, and the debt ratio declines towards its long term target. Utility declines slightly during the convergence process to the steady state but, in the end, there is no difference with the baseline values for any real variable. The long run multiplier is zero. There has been just a shift from future consumption towards present consumption, which raises notably the utility of the current period at the expense of a marginal fall in utility in many future periods.

When the adjustment process implies a sustained rise in the tax burden to reduce again the debt ratio to its initial value, there are more permanent effects. After the impact in the first period described in the previous paragraph, the increase in the tax rate is gradual and the debt ratio rises again slightly above 102% after three periods. The reversal of the utility bonus of the first two periods is swift, as it declines by around 30bp in the following periods, and then proceeds more slowly to stabilize in the long term 17bp below the baseline. Thus, the increase of 1pp in utility obtained in the first year implies a permanent decline in utility of 17bp when it is financed by higher taxation, due to the negative effects on consumption and output in the new steady state. The long run multiplier is -0.5.

The long term implications of a shift of public expenditure towards transfers from productive spending are substantially different from financing the higher transfers with taxes. Although the initial impact is the same, the fall in productive spending reinforces the negative effects on output, which in the long term declines (with a long run multiplier of -1.6), and the hump in the debt ratio reaches a maximum of almost 104% after six periods. It has an impact on utility similar to financing the additional transfers with taxes: there is a shift from future to present consumption and utility, but the steady state values of both variables are not different from the baseline.

1.5.3 Changes in government spending in goods and services

An increase of 1pp in the ratio to GDP of public productive spending in one period, which is reversed slowly in the future, does not have effects in the long run. The initial impact rises consumption, leisure and utility (the three by 0.6pp), together with output growth (short-run multiplier equal to 0.8), and reduces labour supply. The debt ratio even declines slightly because of the denominator effect on the value of the debt from the previous period. The government (and external) deficit also fall (66bp). The immediate effect of this expansionary policy is thus positive on all counts. Thereafter, all these changes are reversed little by little as the debt is forced back to its 100% threshold. At the end of the adjustment process, the baseline is fully restored (the long run multiplier is zero). There have been inter-temporal shifts, but no permanent change with respect to the initial steady state equilibrium.

If the expansionary policy of government spending decided in the first period is financed by taxes to stabilise the debt ratio, the new steady state equilibrium entails a bigger size of the government sector, with permanent changes in the economy with respect to the initial one. The gentle rise of the tax rate does not affect practically the growth rate of output or the deficit, and therefore the debt ratio barely raises 60bp at its peak, while the higher output of the first period remains above the baseline path by 0.7pp for the indefinite future. The long run multiplier is 0.7, practically the same as in the short run. However, the mounting tax burden affects negatively labour supply and consumption, so that the initial increase in utility is reversed and falls permanently below the baseline by 10bp in the final equilibrium.

When the increase in productive spending is compensated by a decrease in transfers, the adjustment process is even smoother than in the previous cases. After the first few periods, deficits and debt, and consumption and labour supply, do not change, while output growth remains above trend for some time, leading in the long run to a level of output which is 1% higher than the baseline. The long run multiplier is bigger than the short run one. Utility levels, in contrast, are the same as in the baseline in the new equilibrium. As in other cases, there is just a shift of utility to the first period, compensated by a small decline in utility in all subsequent periods, until the new steady state is consolidated.

1.5.4 Changes in the target debt ratio

The final exercise in comparative statics of steady state equilibria deals with a change in the long term target of fiscal policy. Starting with a debt ratio of 100%, a relaxation of the target by 10pp allows a cut in taxes or an increase in expenditure. If the intention of such discretionary relaxation is to alleviate the tax burden, the plan is self-defeating. The reduction in the tax rate implemented in the first few periods (80bp) has to be reversed later. The impact of the change in the long term target is null in the first period, as the adjustment only starts in the second one. The effects unfold slowly as the tax rate falls, the government and external deficits rise and the debt ratio drifts up. The alleviation of the tax burden leads to an increase in labour supply and output and in consumption, financed by the rest of the world, and a higher level of current utility. But these variations do not last. As soon as the tax rate has to be raised again to stabilise the debt ratio around the new target, all the previous changes are reversed. In the long run, the tax rate recovers its initial level plus the extra basis points necessary to finance the permanently higher debt ratio to GDP. The debt ratio reaches 110% of GDP, the burden of the debt increases 45bp (given that the interest rate remains constant by assumption at 4.5%), the overall deficit of the government is 40bp higher and the tax rate, like the primary surplus and the trade balance, are 4.5bp higher. During the whole transition process, until the new equilibrium is reached, there is a gain in utility in all periods, but in the end, when the new level of the tax rate is established, the increased burden of the debt entails a small but permanent loss in utility of 4.5bp.

Assuming that the relaxation in fiscal policy seeks increasing transfers or government spending in goods and services produces in the long run a similar outcome to the strategy of reducing the tax rate. In the end, transfers or productive spending have to be scaled down to the starting point of the initial level, but the debt burden has risen. There is the same shift forward of utility during the process of adjustment, at the cost of a minor, even barely noticeable, permanent loss of utility which will be endured by all future generations in the new steady state equilibrium.

2 Passive fiscal policies and automatic stabilisers

Passive fiscal policies in this model mean reactions of the government to exogenous events or shocks with the aim of preserving the sustainability of public finances by implementing the stability rule through the adjustment in fiscal variables. Redistributive and automatic stabilisation effects depend on the structure and levels of fiscal variables, while changes in these variables in order to prevent the economy and public finances drifting away from the steady state equilibrium have the potential to distort temporarily these automatic effects. To analyse the operation of automatic stabilisers we assume that fiscal authorities are satisfied with the initial structure of public finances, but the economy is subject to random shocks in exogenous variables that produce an impact on public finances and require a reaction of policymakers. It should be noted from the beginning that these adjustments in the policy instruments, though marginal, always go in a distinctly pro-cyclical direction so that, according to the stabilisation rule, the required reaction of the authorities to, for example, a negative shock affecting downwards the output of the economy, is to raise taxes and/or cut expenditure, even if it is in a parsimonious way. The authorities can chose which instruments (the tax rate, productive or redistributive spending) to use, and we will examine the consequences of using each one separately. We analyse the impact on the economy of shocks to the various exogenous variables, under the hypothesis that the fiscal authorities react to them by adjusting only one of the instruments at a time to stabilise the debt ratio.

2.1 The impact of exogenous shocks

We consider the impact of shocks in population, technology, preferences and prices by assuming alternatively an increase of 1 pp. in the level of population, productivity or prices, and of 1bp in the parameter α, representing preferences. All these shocks have permanent effects on the levels of exogenous variables, i.e., an increase of one percentage point in productivity rises the level of productivity by 1% forever. We also consider a temporary increase in the interest rate in one period, which falls back to its previous level in the following period. The effects of reductions are symmetric.

2.1.1 Changes in the labour force

An expansion of the population and the labour force has positive scale effects on the sustainability of public finances because the existing debt burden from the past can be shared by more individuals. The increase of 1% in the labour force raises labour supply (which is equal to employment) and the output level. Assuming that the fiscal authorities react adjusting taxation, the subsequent reduction in the tax rate reinforces these initial effects on labour supply. Real variables are permanently higher than in the baseline: output (1pp) consumption (1.2pp), leisure (0.6pp), and utility (1pp). The ratios of productive and redistributive public spending decline by 0.2pp each.

If the authorities react instead raising government spending in goods and services, the effect on output is slightly higher, but lower on labour supply and consumption. There is also a stronger rise in leisure, so that the gain in utility is the same as in the case of a lower tax rate. The ratio G/Y rises 0.3pp matched by a decline of equal size of the ratio I/Y.

Finally, an increase in transfers produces the same effects on individuals as the rise in productive spending, because both are close substitutes from the point of view of the income and utility of an average or “representative” individual. Consumption, leisure and utility are all 1pp higher than in the baseline. Labour supply also rises 1%, but output only 0.8%, as the ratio G/Y declines by 0.2pp, compensated by a similar increase of the ratio I/Y.

2.1.2 Changes in productivity

Changes in productivity have similar effects to those in the labour force on all variables, except on labour supply and utility[8]. An increase of 1pp in productivity followed by a cut in taxes produce a much weaker rise in the labour supply and a negative effect on leisure, which declines because the higher productivity makes leisure more expensive ($\frac{\partial l^*}{\partial k} < 0$). As a consequence, the increase in utility is lower in the new steady state when the labour force rises. If the gains of higher productivity are used to raise expenditure, there is no significant difference in the long term effect on the economy between expanding productive or redistributive spending, whose results are similar to those considered in the previous point.

2.1.3 Changes in preferences consumption-leisure

A permanent increase in the preference of consumption over leisure represented by a rise in the parameter α from 0.8 to 0.81 has substantial effects. The initial impact stimulates labour supply and reduces leisure, rising output by 1.7pp, consumption by 1.3pp, and utility by 1pp. The expansion of output has a denominator effect on the ratios of government spending (0.3pp each), deficit (0.6pp) and debt (2.3pp, of which 1.7pp come arithmetically from the impact on the outstanding debt ratio of 100%).

When this impulse is followed by a policy response of cutting taxes, the tax rate can drop in the new steady state, made possible by the higher output and the decline in the expenditure ratios. The alleviation of the tax burden leads to a further slight decline in leisure and increase in consumption, producing a gain in utility in the long run, even if the steady state output does not expand further.

[8] The reason of this difference is that changes in population or in the transfer level imply a parallel shift in the budget constraint of the consumer decision choice (as shown in equations 6 and 7), while changes in productivity or in the tax rate entail a change in the slope of the budget constraint. The first order conditions for the optimal choice of the consumer requires $\frac{c}{l} = \frac{\alpha}{1-\alpha} \cdot (1-\tau) \cdot k$, which depends on τ and k, but not on n.

If the authorities spend more instead of taxing less, the long run gain in utility is similar, whatever the type of expenditure which is expanded. In both cases, leisure declines less and consumption grows less than when taxes are cut. The main differences are in output, which expands by 2.5pp when productive spending rises versus 1.5pp when transfers are increased, and in the change in the relative ratios of both types of spending. More spending in goods and services leads to a rise in its ratio of 0.5pp, matched by a decline of the same magnitude in the ratio of transfers, while increasing the guaranteed income in the negative linear tax push up by 0.3pp the ratio of transfers and shrinks the ratio of productive spending by the same amount.

2.1.4 Change in the price level

An unexpected increase of 1pp in the price level erodes the real value of the debt and nominal public spending authorised in the budget for the year as well as a rise in taxes on nominal income (and consequently the deficit), so that the debt ratio falls by 1.3pp: 1pp because of the denominator effect on the stock of debt (100%) and 0.3pp because the denominator effect on public spending and the deficit. This has an initial negative impact on consumption, leisure and utility, and a positive impact on labour supply. Despite this additional employment, output declines by 10bp due to the lower productive public spending in real terms and, therefore, nominal GDP raises only 0.9pp. When the tax rate is used to preserve the stability of the debt ratio, it is possible to reduce it in the long run by 0.4pp (because it corresponds to 1% of the ratio of total public spending in GDP), which has positive effects on labour supply and consumption, and a negative one on leisure, yielding a marginal gain in utility and no change in real output. All these are sizeable effects, which are not surprising in view of the fact that the inflationary shock produces a 1pp alleviation of the debt in real terms, which is not dissimilar to a partial default of the same magnitude or an “inflation tax” paid by foreigners, who are the debt holders.

In contrast, if the rise in the price level is used to expand public spending, the “inflation tax” on debt holders is wasted in the end, as there are no long run changes in leisure or consumption and, therefore, no increase in utility, although real output rises (0.3pp) in the case of higher productive spending, while it declines (0.2pp) in the case of more transfers. The changes in the structure of government accounts, depending on which spending item is boosted are the same commented before.

2.1.5 A temporary change in the interest rate

A temporary increase (say of 1pp) of the interest rate just for one period has no lasting effects. When it happens, government interest payments, budget deficit and debt ratios rise by 1pp, since the debt ratio is 100%. In subsequent periods, the process of absorbing the shock unfolds, rising the tax rate or cutting non-financial spending, and reducing the utility level for a number of periods. The deficit ratio has to be reduced with respect to the initial level in order to get the debt ratio back to 100%.

2.2 The effects of automatic stabilisers

Automatic stabilisers effects are the dampening effects that the structure of revenue and spending produces on demand and utility of individuals when there are exogenous shocks. In the stylized economy described above, it is possible to identify such effects calculating the (partial) elasticities with respect to each one of the exogenous variables (labour endowments, productivity, etc.) in the equations showing the optimal decisions of individuals for consumption and leisure. The impact of the shocks will be dampened by automatic stabilisers if such elasticities are lower than one.

In practice, however, the values of the partial elasticities calculated theoretically from the equations may be affected by the reactions of fiscal policy to the same shocks, because as we have just seen, even “passive” fiscal policies require adjustments in the instruments of control of public finances to stabilise the debt ratio. In addition, some variables are not directly observable (like leisure choices or labour skills) and the shocks may be simultaneous. To deal with these possibilities, we estimate through simulations the elasticities of consumption, leisure and utility of individuals with respect to purely real (labour force and productivity) and nominal (price) shocks and to random combinations of them, when the fiscal authorities use only one of the three instruments at a time.

These estimations of different elasticities measure the “effects” of automatic stabilisers. In the next subsection we estimate how the changes in the real growth rate affect public finances and private consumption ratios to GDP, which is an indirect way of gauging the size of automatic stabilisers based only on observable data. We will conduct 100 simulations of 100 years each to estimate these elasticities. The structure of the economy is the same we had before and the interest rate will be kept constant at r=4.5%. In the rule of stability, we use the parameters u=0.05 and v=0.5, as in the exercises of the previous analysis of fiscal multipliers. In this section, the economy is subject to random shocks to the growth rates of population, productivity and prices. The shocks have a uniform distribution in a range around their central values of $\pm 0.02\ for\ \eta, \pm 0.01\ for \xi, and \pm 0.02\ for\ \pi$.

2.2.1 Elasticities of consumption, leisure and utility to exogenous real and nominal shocks

The elasticities of consumption, leisure and utility with respect to the labour endowment are all around 0.69 with standard deviations of 0.05 for utility and consumption and 0.02 for leisure. The values of these elasticities are the same for the three instruments of adjustment. When the shocks to the economy come from productivity changes, the elasticity of consumption does not change, because labour endowments and productivity affect consumption demand in the same way, as shown in equation [6], whereas the elasticity of leisure with respect to variations in productivity is now negative (around -0.3, with standard deviation 0.02). The elasticity of utility, which is the weighted average of those of consumption and leisure with weights α and (1-α), respectively, is 0.49 with standard deviation of 0.03. It is worth noting that the changes in these elasticities are not significant when the fiscal tools of stabilisation change.

When considering shocks to prices, we have seen that their influence is via the impact on public spending in goods and services and in transfers, as well as on the value of existing debt, measured in real terms, and on taxes on nominal income. The estimated elasticities of consumption, leisure and utility are around -0.3 (with a standard deviation of 0.02), so that nominal shocks create noise which reduces consumption, leisure and utility of individuals. This noise is compounded when fiscal authorities react mechanically adjusting the tax rate in the opposite direction to price surprises in order to stabilise the debt ratio, as can be seen in the multiplicative interaction of prices and taxes in equations [6] to [8]. The distortionary effects of unpredictable price shocks on short-term stability will be further clarified below when examining semielasticities.

Finally, the estimated elasticities are robust to the simultaneous impact of the three types of shocks, although the standard deviations are approximately doubled.

2.2.2 The size of automatic stabilisers estimated through semielasticities

An indirect way to look at the operation of automatic stabilisers in practice is to focus on how government revenue and expenditure, as well as private disposable income and consumption, react to economic fluctuations. To estimate the size of the automatic stabilisers embedded in the structure of public finances with observable data, we introduce a random combination of the shocks mentioned before shocks and allow the stabilisation rule to function, using separately the different policy instruments. We consider 100 simulations of shocks in 100 periods each time, and estimate the semielasticities (i.e., the elasticities minus one times the ratio of the variable to GDP)[9] of revenue, expenditure and the budget balance through simple linear regressions of the changes in the ratios to GDP of the variables ($\Delta\left(\frac{v}{y}\right)$) on the rate of real output growth of the economy ($\frac{\Delta y}{y}$). This is a simple way to present the stabilising effects of fiscal policies, which has also the advantage of preserving the additivity of the components of the budget constraint, meaning that the semielasticity of total expenditure is equal to the sum of the semielasticities of its components, and the semielasticity of the budget balance is the difference of the semielasticities of revenue and expenditure. We also report the semielasticity of consumption, which is equal to the semielasticity of disposable income, which in its turn is equal to the difference of the semielasticities of taxes and transfers. The estimates reported can be checked with the spreadsheet *Semielasticities* accompanying this document.

When there are shocks to the labour force or productivity, the results of the repeated regressions are robust and very similar whether the instruments used by the government are the tax rate or transfers. In contrast, the utilization of productive spending to stabilise the debt ratio reduces significantly the estimated semielasticity of this variable and, consequently, also the semielasticity of the government budget

[9] Notice that if v is a variable and y is output:

$$\frac{\Delta(v/y)}{\Delta y} = \frac{\Delta v}{y} - \frac{\Delta y}{y} \cdot \frac{v}{y} = \left[\frac{\Delta v}{v} - \frac{\Delta y}{y}\right] \cdot \frac{v}{y} = \left\{\frac{\Delta v/\Delta y}{v/y} - 1\right\} \cdot \frac{v}{y} \cdot \frac{\Delta y}{y},$$

where the expression in curled brackets {} is the semielasticity, v/y is the ratio to output and Δy/y is the rate of growth of output.

balance, as shown in the table of Annex 2, together with the respective coefficients of correlation (in the second row).

There are several stylized remarks and tentative conclusions from the table with the estimates in Annex 2 which are worth pointing out:

1. The estimates reported depend on the particular history of shocks realized in each simulation and may underestimate the potential short run effect of automatic stabilisers. The reason is that a purely mechanical reaction with a predetermined policy instrument to whatever shocks hit the economy is probably a suboptimal approach to smooth short run cyclical fluctuations.

2. Information about the nature of the shocks, and a flexible response with the most suitable instruments, should improve the stabilisation effects of fiscal policies in the short run, without jeopardising long-term sustainability. In the absence of timely information, using a combination of instruments seems to be a better option than relying on only one of the three instruments

3. The semielasticities are not sensitive to the changes in the target debt ratio[10], or in the parameters (u,v) regulating the speed of adjustment (within the limits that define the stability conditions of the dynamic system), or in the size of the shocks. This has important policy implications because it shows the compatibility of the short-term operation of automatic stabilisers with a steady policy of fiscal consolidation aiming at reducing forcefully the debt ratio..

4. Taxes do not have a semielasticity significantly different from zero, reflecting the fact that a proportional tax on output should show by definition a unitary elasticity with respect to output.

5. Both types of non-financial expenditure have a semielasticity very much in line with the size of their respective ratios to GDP, reflecting the fact that, when the

[10] The only change when we repeat these simulations imposing b*=20 in the long run, instead of remaining at the initial debt ratio b=100, is that the semielasticity of interest payments is now -0,03, i.e. half of the previous one, due to the fact that the debt ratio is no longer stationary. All other semielasticities are the same.

policy is to keep these ratios stable, their elasticity to output growth should be zero, and therefore, their semielasticity equal to the value of their ratio to GDP with negative sign. The correlation with growth fluctuations is significant, except when productive spending is the tool of stabilisation.

6. If the instrument used is government spending in goods and services, the distortionary impact on the economy is more severe than with taxes, because its “multiplier” on the demand side is higher and the impact on the supply is also procyclycal, as shown before (in the comparative statics exercises). Consequently, the estimated values of the semielasticity and the correlation coefficient are very low.

7. Using the guaranteed minimum income as the policy instrument reduces sharply the coefficient of correlation, but does not affect the estimated value of its semielasticity as much as in the case of productive spending. This is because the “procyclical” adjustment of transfers, e.g. cutting the minimum guaranteed income when there is a fall in output growth, has contractive effects only on the demand side (reducing consumption and utility), but expansionary effects on the supply side (increasing employment and output).

8. The semielasticity of interest payments is stable at -0.06 in all cases and showing a very high coefficient of correlation (above 0.9), due to the fact that interest rates are constant and the debt ratio to GDP is stationary around the initial level (100%).

9. The semielasticity of the budget balance is exactly the linear combination of the semielasticities of revenue and expenditures, by construction, and the correlation coefficient is not as large as those of the components of the budget constraint.

10. The estimated average size of automatic stabilisers in this model is reasonable, but their effects are not always systematic, as evidenced by the low value of the correlation coefficients for the variables that are used as policy instruments.

11. The semielasticity of private consumption is always slightly above 0.2 which is in line with its elasticity to income (0.75 with standard deviation 0.03) times its weight in GDP (80%). Nevertheless, the coefficient of correlation is much lower when the policy instruments used affect disposable income (the tax rate or the minimum income) than when it is spending in goods and services. Hence, if the short-term objective of automatic stabilisers is to smooth consumption fluctuations, productive spending should be the primary instrument of choice to react to cyclical fluctuations in order to keep public finances on track in the long run.

12. Turning now to the impact of random, unpredictable, price shocks in the short-term, the estimates of semielasticities suggest that using the tax rate as instrument in the presence of a price shock is destabilising, while using public spending in goods and services is not stabilising, and only the adjustment of the minimum income seems to have significant stabilising effects on private consumption (a high value of the estimated semielasticity), which are nevertheless extremely unsystematic (a very low correlation coefficient).

3 The Reactions of financial markets to changes in the debt ratio and the probability of default

Financial markets reactions to fiscal policies depend on their assessment of the sustainability of public finances. Although such reactions are notoriously unpredictable and varying in time, a simple formalization via a rule of thumb can help in understanding the implications of changing market sentiment for the sustainability of public finances. We will assume that interest rates respond to debt developments by applying a risk premium to the difference between the last observed value of the debt ratio and a moving average of this ratio ratio in the last five periods. On the other hand, when the debt ratio is low and stable or steadily decreasing, it is assumed that there is a floor for the interest rate equal to the minimum interest rate observed in the past twenty years.

Under these hypotheses, it is possible to simulate the model in order to estimate the probabilities of default associated to different debt targets and choices of implementation instruments by policy makers, when the economy is subject to shocks to population, technology, preferences, and prices, with a uniform distribution around (plus or minus) the mean values of the respective parameters and inflation rate. The maximum amplitude of the shocks in each direction is given by:

{η=0.02; ξ= 0.01; α=0.01;π=0.02}

These maximum values of the shocks correspond approximately to one standard deviation of those estimated for the Spanish economy in the period 2000-2013. The values of the parameters are kept initially at u=0.05 and v=0.5 as before. Each time, 100 simulations of the performance of the economy are repeated. In each simulation, it is checked whether the economy has defaulted at a 25-year horizon. The probability of default (in percentage points) is estimated to be the number of simulations (between 0 and 100) in which the economy has defaulted, and the associated standard deviation is also computed from this number[11]. The focus of the simulations is on the probability of default for alternative target debt ratios (between 40% and 120%), when the reaction of interest rates to debt ratio developments takes different values (between 1 and 15 basis points). The table in Annex 3 shows the results of these exercises. In each cell, the first number gives the estimated probability of default, and the second one the standard deviation of the sample. The estimates of the default probabilities can be checked with the spreadsheet *Default probabilities* accompanying this document.

It is possible to draw some conclusions about probabilities of default from these simulations:

1. A sustained pace of fiscal consolidation with an ambitious target is the best insurance against the possibility that adverse developments in the economic environment lead to a debt crisis. Probabilities are higher the more relaxed the target debt ratio and the longer the time horizon considered. Keeping a high debt ratio for long, and even more rising it steadily, increases the fragility of

[11] Let p (between 0 and 1) be the probability of default. The associated standard deviation is given by the formula $[p \cdot (1-p)]^{0.5}$.

public finances to negative shocks as time goes by. Low probabilities of default at a short horizon rise to approach 1 when the period considered is extended from 25 to 50 and 100 years.

2. Probabilities are higher when productive spending is the instrumental variable used by fiscal authorities to respond to shocks. As argued before, the reason is that its “multiplier” impact on output, on consumption and labour supply is stronger than the effects of changing the tax rate or the minimum income. Consequently, the “procyclical” adjustments necessary to implement the stability rule when there is a negative shock can further depress output and drive up the debt ratio and interest rates, triggering a debt crisis which leads to default. Using taxes and transfers is more effective because of their positive supply effects on consumers´ choice.

3. Probabilities rise when the speed of reaction of policy adjustments to debt developments (the size of the parameter u in the rule) is lower, because a slow policy reaction may not be enough when confronting a sequence of negative shocks that reduce growth and increase the debt ratio, further pushing interest rates up, in a negative feedback loop with even lower output and higher debt ratios. Nevertheless, for a high and increasing debt ratio (from 100% to 120% in the table of Annex 3), a higher speed of reaction is associated to higher default probabilities, whatever the instrument used. The reason is again in the mechanical reactions of the three types of agents in the economy: a strong adjustment of fiscal policy to an adverse shock generates a negative impact on output and a rise in the debt ratio, which leads to higher interest rates and can trigger a debt crisis.

4. Probabilities increase in a non-linear way with the size of the reaction of interest rates to debt ratio developments. Taking into account that market sentiment can be volatile rather than stable when a crisis starts unfolding, the non-linearity of the probabilities estimated is, if anything, even more pronounced than appears in the estimates from these simulations, where financial market participants behave steadily and the speed of reaction of interest rates to rising debt ratios remains constant until the very end of a debt crisis.

5. As already noted in the analysis of automatic stabilisers, a graduated reaction that takes into account the nature of the shocks afflicting the economy and selects the most appropriate instrument in each situation would prove a more effective reaction than an automatic reaction.

4 Simulating alternative paths of the deficit and debt ratios under the rules of the Spanish Organic Law of Budgetary Stability and Financial Stability

The examples given in the previous section strongly support the idea that fiscal prudence requires ambitious targets to reduce steadily and consistently the debt ratio, not least for stability reasons. Fiscal relaxation increasing the utility of the current taxpayers not only shifts to the future the cost of the debt burden and is unfair for those who cannot vote now, it also jeopardises economic stability and risks triggering a financial crisis in unpredictable time. That is one of the rationales for establishing at the maximum level of legal commitment fiscal rules that preserve fiscal discipline and protect future taxpayers.[12]

4.1 The rules of fiscal discipline in the EU and in Spain

Perhaps the simplest rule of fiscal discipline is to keep the budget in balance or in a small surplus. However, when the economy is subject to unpredictable shocks, to balance the budget in each period interferes with the full operation of automatic stabilisers built into the existing provisions that determine government revenue and expenditures. If such provisions have to be changed in response to contemporaneous shocks in order to balance the budget immediately, raising for example taxes when there is a recession, the stabilising role of a given income tax with some progressivity will be severely distorted. Perhaps, the simplest modification of the balanced budget rule to make it compatible with the operation of automatic stabilisers is to react with a lag, allowing shocks to work through the existing provisions, but changing the relevant

[12] See Wyplosz (2012) for a detailed analysis on the motivation for fiscal rules and historical experiences.

policy variables for the future so that the budget does not drift away very much from balance, and thus the sustainability of public finances is guaranteed. The simulation model described before can be used to understand the dynamics of the debt ratio under these regulations. One important point to stress is that the current regulations rely heavily on the concepts of output gap and structural budget balance, while in the economy presented in these notes, there is no unemployment, the output gap is always zero and the structural balance is the actual one. It would be easy to introduce these concepts in the model to make it superficially more “realistic”, but this would be at the cost of increasing the difficulties to understand the problems that fiscal authorities face to preserve economic stability.

In the EU regulations of the Stability and Growth Pact, when there is an excessive deficit, meaning that there is a government deficit higher than 3% of GDP (d_t<-3%), there are specific provisions to reduce it below this reference value, first, and to bring it further down later. In addition, when the debt ratio is above a reference value of 60% (b_t>60%), it is required that the debt ratio declines by at least 5% of the difference between its current value and the reference value. When, instead, d_t>-0.5% and b_t<60%, government finances are in a close to balance or in surplus position and the EU rules do not impose constraints on national fiscal policies. The detailed regulations are complex,[13] but for our present purpose it is enough to understand the thrust of them. To represent EU rules, the required adjustment in the debt ratio when b_t>60% is:

$$b_t - b_{t-1} = -0.05 \cdot (b_{t-1} - 60)$$

We can replace b_t in this expression from the government budget constraint:

$$b_t = \frac{b_{t-1}}{1 + g_t} - d_t$$

to get the implicit value for the deficit target in year t (d_t*):

$$d_t^* = (0.05 - 0.95 \cdot g_t) \cdot b_{t-1} - 0.05 \cdot 60 \cdot (1 + g_t)$$

[13] See Frayne and Riso (2013) for a detailed and updated description of the EU fiscal regulations.

This rule is binding until the budget is balanced. Once the government financial position is close to balance or in surplus, the target is to keep that position.

In Spain, the fiscal rules contained in the Organic Law of Budgetary Stability and Financial Sustainability (SL, in short) are more demanding.[14] The basic principle of budget stability requires that the budget is not in deficit ($d_t \geq 0$), according to article 11 of the SL. Furthermore, in the transition period until the debt ratio is below the reference value of 60%, the SL sets up three requirements in its transitory provision 1:

1. Government spending, excluding interest payments, should grow at a lower rate than the rate of growth of real output: $\frac{G_t+P_t}{G_{t-1}+P_{t-1}} < (1+\eta)\cdot(1+\xi)$
2. If Spain is subject to an Excessive Deficit Procedure, the deficit should be cut in line with the EU Council recommendations for its correction, and by at least 0.8 percentage point per year otherwise: $d_t > d_{t-1} + 0.8$
3. Whenever the rate of growth of real output is 2% or more, or in case there is a net increase in employment of 2% or more, the debt ratio should be cut by at least 2 percentage points: $(1+\eta)*(1+\xi)>1.02 \Rightarrow b_t - b_{t-1} < -2$.

4.2 Calibration of the model and starting point of the simulations

To simulate the paths of deficit and debt ratios, the initial parameterization has been adjusted to reproduce similar variables of the Spanish economy:

$$\{n_0 = 1; k_0 = 1; \eta = 0.5\%; \xi = 1.5\%; \alpha = 0.825;\ r = 4\%, \pi = 1.8\%, b_0 = 98.5\%\}$$

The government reaction function is calibrated to follow the provisions in the SL in terms of debt reduction and deficit adjustment requirements. We assume first that the years 2014-2017 follow the projections of the Updated Stability Programme of Spain

[14] See Hernández de Cos and Pérez (2013) for a description and analysis of the main features of the law.

(SP for short).[15] The growth rate of GDP and the equilibrium values of the fiscal variables are:

Table 1: Projections under the 2014-2017 Stability Programme

Year	Expenditure	rb	g	G/Y	I/Y	S	d	B
2014	44.0	3.5	1.2	24.0	16.5	-2.0	-5.5	99.5
2015	43.0	3.6	1.8	23.1	16.3	-0.6	-4.2	101.7
2016	41.8	3.7	2.1	22.1	16.0	0.9	-2.8	101.5
2017	40.1	3.8	3.0	20.9	15.4	2.7	-1.1	98.5

Source: Updated Stability Programme

From 2018 onwards, the rules described above are operating fully. The steps to calibrate the model to the starting values of the SP are the following:

1. The value of nominal and real GDP (Y and y) in 2014 are equal and fixed in trillion euros at the value for nominal GDP projected by the SP. The rates of real growth and inflation (and therefore the paths for y and Y) are those projected in the SP until 2017 and thereafter, they are constant at 2% and 1.8%, respectively.

2. The interest rate increases gradually to 4% in later years, and remains constant thereafter. The values of deficit and debt in 2014-2017 are determined by the ratios to GDP in the SP and the path of Y given in 1.

3. The tax rate is assumed to be the current revenue to GDP ratio of the SP (τ=38.5% in 2014, rising to 38.8% and 39% in 2015 and 2016, and remaining at that level in 2017)

15 The Updated Stability Programme 2014-2017 is available in the web page of the Ministry of Economy and Competitiveness: http://www.mineco.gob.es/stfls/mineco/comun/pdf/Estabilidad_2014_2017.pdf

4. The ratios of expenditure to GDP of the SP, together with nominal GDP of step 1, determine the values of G and P.

5. The path of n is fixed at n=1 in 2014, and growing as employment in 2015-2017. Later on, the rate of growth is constant at 0.5%. The path of productivity (k) is fixed by real growth less employment, starting from k=1 in 2014.

6. The non-observable path of leisure is determined by calibrating the parameter α in the consumption function so that *l* in equation [7] satisfies also its value derived from the production function: $l^{*} = n - \frac{y-G/p}{k}$.

4.3 Alternative paths of the deficit and debt ratios

The SP projects a government deficit of 1.1% of GDP and a debt ratio of 98.5% in 2017. According to the Transitory provision 1 of the SL, the period allowed to reduce the debt ratio below 60% ends in 2020. It is extremely unlikely, if not unfeasible, to comply with this deadline. The SL itself envisages the possibility of redefining in 2015 and 2018 the path of the debt ratio to attain this target. If the transitory period is extended beyond 2020, there are two related questions to consider which the model can help answer:

- What could be a reasonable new deadline?
- Should the rules imposed in the transitory period be changed as well?

4.3.1 It would be sensible to extend the deadline beyond 2020

The most stringent requirement of Transitory provision 1 is to limit the growth of non-financial spending to a rate below the growth rate of real GDP. With a ratio of non-financial spending to GDP of more than 40% and an inflation rate of 1.8%, fulfilling this requirement would bring down this ratio by 0.7 percentage points per year for an extended period of time. If the tax rate is kept at the level of 2017 envisaged in the SP (39%) the debt ratio could come below 60% in 2025, with a decline of more than 9 percentage points of GDP in the ratio of non-financial spending to GDP between 2014 and 2025, and a primary surplus of more than 7% and an overall surplus of 5% in that year. It is doubtful that this scenario with high and persistent budget surpluses could be realised (See charts from simulation 1 in Annex 4).

4.3.2 It would be sensible to change Transitory provision 1 of the SL

In case the transitory period to reduce the debt ratio below 60% is extended beyond 2025, fixing a new deadline depends on the maximum budget surplus ratio that is considered feasible and on the number of years to reach and maintain sizeable surpluses. For example, keeping the tax rate at 39% only until an overall surplus is realised in 2019, while respecting still the limit to the growth rate of non-financial spending fixed in Transitory provision 1, would produce a budget surplus slightly below 1% of GDP in that year. If this budget position is maintained thereafter, the debt ratio will be below 60% in 2028, even allowing for a gradual decline of 6 percentage points in the tax rate. This reduction would be the consequence of a severe decline by more than 12 percentage points in the ratio to GDP of non-financial spending between 2014 and 2028, which seems difficult to attain. This example suggests that it would be sensible not only to extend the length of the transitory period beyond 2020, but also to change the requirement limiting so stringently the growth of non-financial spending (See charts from simulation 2 in Annex 4).

4.3.3 A sensible constraint would be to keep a budget surplus from 2018 onwards

The restriction on the growth of non-financial government spending could be circumscribed to periods in which there is no overall surplus in government accounts, a provision which would still keep the debt ratio on a reasonably steep path of decline. Allowing government non-financial spending to bear the full weight of adjustment after 2017 in order to reach a small budget surplus of half a percentage point of GDP, while keeping constant the tax rate at the level of 39% attained that year, the ratio to GDP of non-financial spending would still need to decline by 1.3 percentage point. If this adjustment is front-loaded to 2018, which seems feasible being smaller than the one projected for 2017, the spending ratios could rise gradually in the following years to reach the same level of 2017 by 2030, when the debt ratio would be again below the 60% reference value. These paths seem to be feasible as a baseline. To illustrate the sensitivity of the debt ratio path to output growth, the year in which the ratio falls below the reference value of 60% would be 2027, if the average growth rate is one percentage point higher than the baseline (2%), and 2034, if it is one percentage point lower (See charts from simulation 3 in Annex 4).

5 Conclusions

Traditionally, the elemental arithmetic of sustainability of public finances calculations is based on assumptions of constant growth and interest rates, the initial debt ratio and projections of the primary balance. Fiscal authorities are supposed to be in command of the primary budget balance, while the real economy and financial markets do not react to fiscal policies.

The analysis of these two more complex issues proceeds performing simulations of the model, which despite being extremely simple in its structure and patently unreal in its assumptions, produces realistic patterns of adjustment in observable variables, allowing to establish the logical connection between stylised facts associated to statistical regularities (e.g., the semielasticities of fiscal variables) and the underlying assumptions of the model.

This note introduces into these calculations a minimum rationality of choice by individuals as workers, consumers, taxpayers and recipients of transfers, together with a distorting nature of social protection systems through taxes and transfers, a productive component of government spending and a mechanical linear behaviour of financial market participants. These additional elements of structure, naïve as they are, provide a rich enough environment to illustrate the type of constraints under which fiscal policies operate in practice to pursue their objectives. This setting allows better understanding of the issues related to financial sustainability in a dynamic economy subject to random shocks.

From a theoretical perspective, the first and most important and practical takeaway on financial sustainability is a trivial one, although often ignored in practice by policymakers: the assessment of economic policy “measures” requires a fully-fledged specification of its future fiscal implications. Section 2 has analysed at considerable length the outcomes of active fiscal policies. It is worth repeating that the immediate impact of many changes is reversed in the longer term, with the result of temporary shifts in output, consumption or utility between individuals at different points in time. When the changes are not reversed, they always imply a temporary gain or loss in utility, at the cost of a permanent loss or gain, respectively (e.g. relaxing the target for the debt ratio would lead to higher burden of the debt materialised in the need to deliver a higher primary balance and external trade surplus).

Second, the instruments used by fiscal policies to stabilise the debt ratio in the presence of exogenous shocks have different effects, depending on whether they impact directly output (spending in goods and services) or the individuals´ disposable income (taxes and transfers). Understanding the differential impact is useful to assess later the operation of automatic stabilisers and the probability of debt default in a dynamic context. It is important to repeat here a couple of “stylised facts” derived from the simulations. One of them is that using government spending in goods and services may be more efficient to smooth private consumption through cyclical fluctuations, but it is also more risky from the point of view of preserving the sustainability of public finances, so that there is a certain trade-off for the authorities to choose. Furthermore, semielasticities are not sensitive to the changes in the target debt ratio and in the parameters (u,v) regulating the speed of adjustment to attain the target. This shows that there is no substantial trade-off between short and long run policies of stabilisation.

Apart from the abstract analysis of the sustainability issue, the basic model developed in the first sections of the note can also be used to provide some rough calculations about the paths of government deficits and debt implied in the requirements of the current Spanish Organic Law of Budgetary stability and Financial Sustainability. Such calculations are of a purely arithmetic nature, in the sense that they do not depend on the structure of the model, but just on the budget constraint of the government, the starting point of the variables and the operation of the rules of budgetary stability in alternative ways. The main conclusions of this arithmetic exercises are:

1. Given the projections for public debt outlined in the Updated Stability Programme (98.5% of GDP in 2017), the target of the Organic Law on Budget Stability and Financial Sustainability is unlikely to be met by 2020.
2. The authorities should take advantage of the possibility of redefining in 2015 the required path to reach the 60% of GDP target for the debt ratio, extending the transitory period beyond 2020.

The application of the first transitory provision of the law, limiting the growth of non-financial spending, appears too restrictive and could somehow be reformulated without jeopardising the sustainability of public finances. One possibility would be to circumscribe its application to periods in which there is no overall surplus in government accounts, a provision which would still keep the debt ratio on a reasonably steep path of decline.

6 References

Barro, Robert Joseph. 1979. On the Determination of the Public Debt. *Journal of Political Economy* 87: 940-971.

Diamond, Peter A. 1965. National debt in a neoclassical growth model. The American Economic Review, 55(5):1126–1150.

Van Ewijk, Casper, Nick Draper, Harry ter Rele and Ed Wesrerhout. 2006. “Ageing and the sustainability of Dutch public finances”. CPB Special Publication 61.

Frayne, Christine and Stéphanie Riso. 2013. Vade mecum on the Stability and Growth Pact. *European Economy. Occasional Papers*. 151.

Herndon, Thomas, Michael Ash and Robert Pollin. 2014. Does High Public Debt Consistently Stifle Economic Growth? A Critique of Reinhart and Rogoff. *Cambridge Journal of Economics* 38 (2): 257-279.

Hernández de Cos, Pablo and Javier J.Pérez. 2013. La nueva ley de estabilidad presupuestaria. *Boletín Económico*, abril 2013, Banco de España: 65-78.

Kydland, Finn E. and Edward C. Prescott. 1982. Time to Build and Aggregate Fluctuations. *Econometrica* 50 (November 1982): 1345-1370.

Marín, José. 2002. “Sustainability of Public Finances and Automatic Stabilisation under a Rule of Budgetary Discipline”. European Central Bank Working paper nº 192, November 2002.

Musgrave, Richard. 1986. *A Reappraisal of Financing Social Security*. In R. Musgrave (ed.), Public Finance in a Demographic Society, Vol. II., Wheatsheaf Books: 103-122.

Reinhart, Carmen M. and Kenneth S. Rogoff. 2010. Growth in a Time of Debt. *American Economic Review* 100 (2): 573–78.

Wyplosz, Charles. 2012. *Fiscal Rules: Theoretical Issues and Historical Experiences*. In Fiscal Policy after the Financial Crisis, 2012: 495-525, from the National Bureau of Economic Research.

ANNEX 1: MATHEMATICAL NOTES

1.- The consumer´s optimization problem

$Max\ U(c,l) = c^{\alpha} \cdot l^{1-\alpha}$,

subject to: $p \cdot c = (1-\tau)[p \cdot k \cdot (n-l) + G] + I$

1. $L = c^{\alpha} \cdot l^{1-\alpha} - \lambda \cdot \{p \cdot c - (1-\tau)[p \cdot k \cdot (n-l) + G] - I\}$
2. $\frac{\partial L}{\partial c} = \alpha \cdot \frac{U}{c} - \lambda \cdot p = 0$
3. $\frac{\partial L}{\partial l} = (1-\alpha) \cdot \frac{U}{l} - \lambda \cdot (1-\tau) \cdot p \cdot k = 0$
4. $\frac{\partial L}{\partial \lambda} = -p \cdot c + (1-\tau)[p \cdot k \cdot (n-l) + G] + I = 0$
5. From 2 and 3: $\frac{\alpha}{1-\alpha} \cdot \frac{l}{c} = \frac{1}{(1-\tau)\cdot k} \rightarrow c = \frac{\alpha(1-\tau)}{1-\alpha} \cdot k \cdot l$
6. Replacing 5 into 4: p$\cdot \frac{\alpha(1-\tau)k}{1-\alpha} \cdot l = (1-\tau)pkn - (1-\tau)pkl + (1-\tau)G + I$

$$l^{*} = (1-\alpha) \cdot \left[n + \frac{G}{pk} + \frac{I}{pk(1-\tau)}\right]$$

7. Replacing 6 into 5: $c^{*} = \alpha(1-\tau)\left[kn + \frac{G}{p} + \frac{I}{p(1-\tau)}\right]$
8. Replacing 6 into the production function:

$$y^{*} = k\left[n - (1-\alpha)\left(n + \frac{G}{pk} + \frac{I}{pk(1-\tau)}\right)\right] + \frac{G}{p} \rightarrow$$

$$y^{*} = \alpha \cdot \left(k \cdot n + \frac{G}{p}\right) - \frac{1-\alpha}{1-\tau} \cdot \frac{I}{p}$$

9. In the steady state, $G_t = \tau_t \cdot Y_t - I_t - (r-g) \cdot B_{t-1}$ and replacing 8 into this equation:

$$G^{*} = \frac{1}{1-\tau \cdot \alpha} \cdot \{[\tau \cdot \alpha \cdot p \cdot k \cdot n] - (r-g) \cdot B_{t-1}\} - \frac{I}{1-\tau}$$

Notice that $G^{*}(\tau = 0) < 0\ and\ G^{*}(\tau = 1) < 0$, so that G* attains a maximum for some tax rate between 0 and 1.

10. Replacing 9 into the 8:

$$y^{*} = \alpha kn + \alpha\left\{\frac{\tau\alpha kn}{1-\tau\alpha} - \frac{r-g}{1-\tau\alpha} \cdot \frac{B_{t-1}}{p}\right\} - \frac{\alpha I}{p(1-\tau)} - \frac{1-\alpha}{1-\tau} \cdot \frac{I}{p}$$

$$y^{*} = \frac{\alpha}{1-\tau\alpha} \cdot \left[kn - \frac{(r-g)B_{t-1}}{p}\right] - \frac{I}{(1-\tau)p}$$

Output is positive for a zero tax rate and declines for high values of the tax rate.

11. Replacing 7 and 6 into the utility function:

$$U(c,l)=\left[\alpha(1-\tau)k\left\{n+\frac{G}{pk}+\frac{I}{pk(1-\tau)}\right\}\right]^{\alpha}\cdot\left[(1-\alpha)\left\{n+\frac{G}{pk}+\frac{I}{pk(1-\tau)}\right\}\right]^{1-\alpha}$$

$$U(c,l)=\left[\frac{\alpha(1-\tau)}{1-\alpha}k\right]^{\alpha}\cdot(1-\alpha)\{n+\frac{G}{pk}+\frac{I}{pk(1-\tau)}\}$$

2.- Stability analysis

Consider the system of the form $x_{t+1}=A\cdot x_t+\zeta$:

$$\begin{bmatrix} b_{t+1} \\ s_{t+1} \end{bmatrix}=\begin{bmatrix} \frac{1+r}{1+g}-u & -(1-v) \\ u & (1-v) \end{bmatrix}\cdot\begin{bmatrix} b_t \\ s_t \end{bmatrix}+\begin{bmatrix} \left(u\cdot b^*-v\cdot s^*\right) \\ -\left(u\cdot b^*-v\cdot s^*\right) \end{bmatrix}$$

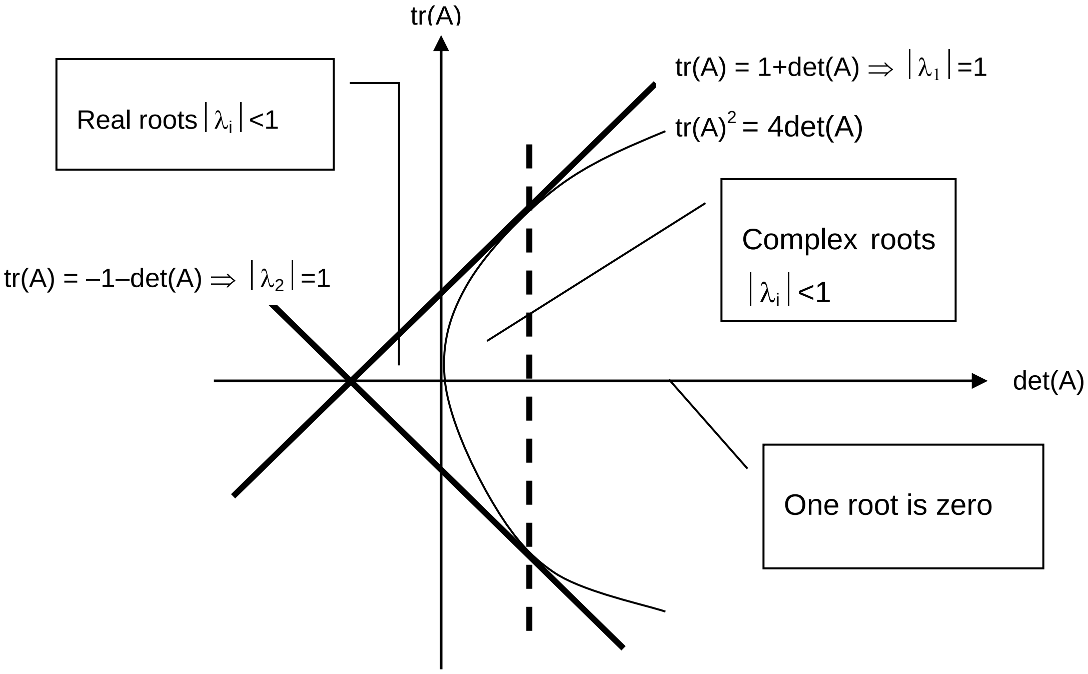
The stability properties of the system equilibrium (b*,s*) and the dynamics of convergence (or divergence) of the debt and primary balance ratios to this equilibrium position depend on the roots of the characteristic equation: $\lambda^2-tr(A)\cdot\lambda+\det(A)=0$, which are:

$\lambda_{1,2}=0.5\cdot\{tr(A)\pm[tr(A)^2-4\cdot det(A)]^{1/2}\}$

where $tr(A)=[(1+r)/(1+g)]-u+(1-v)$ and $det(A)=[(1+r)/(1+g)]\cdot(1-v)$.

The conditions for the equilibrium to be globally stable, guaranteeing convergence from any initial position, require that both roots of this equation are smaller than one in modulus. It is easy to show that: $\{v > [(r-g)/(1+r)]\}$ and $\{u > [(r-g)/(1+g)]\cdot v\} \Rightarrow |\lambda_i| < 1$, i=1,2.

- $det(A) < 1 \Rightarrow v > [(r-g)/(1+r)]$
- $tr(A) < 1 + det(A) \Rightarrow u > [(r-g)/(1+g)]\cdot v$



1. $\{v > [(r - g)/(1+r)]\}$ and $\{u > [(r - g)/(1+g)]\cdot v\} \Rightarrow |\lambda_i| < 1$, i=1,2

   Proof: Let $\eta=(1+r)/(1+g)$, $v=[r-g+\varepsilon(1+g)]/(1+r)$ and $u=[(r-g)/(1+g)]\cdot v+\delta$, with ε and δ positive and small real numbers. Then:

   $\det(A)=\eta\cdot(1-v)=1-\varepsilon$ and $tr(A)=1+\eta-(u+v)=1+\eta\cdot(1-v)-\delta=1+1-\varepsilon-\delta=2-(\varepsilon+\delta)$.

   $\lambda_{1,2} =0.5\cdot\{2-(\varepsilon+\delta)\pm[\{2-(\varepsilon+\delta)\}^2-4\cdot(1-\varepsilon)]^{1/2}\}=0.5\cdot\{2-(\varepsilon+\delta)\pm[(\varepsilon+\delta)^2-4\delta]^{1/2}\}$

2. $u > [(1+r)/(1+g)] + (1-v) -2\cdot[\eta\cdot(1-v)]^{1/2} \Rightarrow \lambda_i$ are complex

   Proof: Let $\eta=(1+r)/(1+g)$. Then:

   $u > \eta + (1-v) -2\cdot[\eta\cdot(1-v)]^{1/2} \Rightarrow$

   $[\eta-u+(1-v)]^2 -4\cdot\eta\cdot(1-v) = tr(A)^2-4\cdot\det(A) < 0 \Rightarrow \lambda_i$ are complex.

3. $u = [r/(1+g)]\cdot v$ and $v \geq [2(r+g)/(1+r+g)] \Rightarrow tr(A) - 4\cdot\det(A) > 0$

   Proof: $u = (r\cdot v)/(1+g) \Rightarrow u+v=[(1+r+g)/(1+g)]\cdot v \Rightarrow$

   $tr(A)^2-4\cdot\det(A)=[1/(1+g)]^2\cdot[(1+\Psi)^2 -4\cdot\Psi-4\cdot r\cdot g\cdot(1-v)])=[1/(1+g)]^2\cdot[(1-\Psi)^2 -4\cdot r\cdot g\cdot(1-v)]$,

   where $\Psi=(1+r+g)\cdot(1-v)$. Then, assuming $v = [2(r+g)/(1+r+g)]$, $1-v=[1-(r+g)]/[1+(r+g)]$:

   $tr(A)^2-4\cdot\det(A)=[1/(1+g)]^2\cdot\{(r+g)^2-4\cdot r\cdot g\cdot[1-(r+g)]/[1+(r+g)]\}>0$

   because $r>g>0$, $[1-(r+g)]/[1+(r+g)]<1$ and

   $(r+g)^2=r^2+g^2+2\cdot r\cdot g=(r-g)^2+4\cdot r\cdot g>4\cdot r\cdot g>4\cdot r\cdot g\cdot[1-(r+g)]/[1+(r+g)]$

ANNEX 2. SEMIELASTICITIES TO REAL SHOCKS

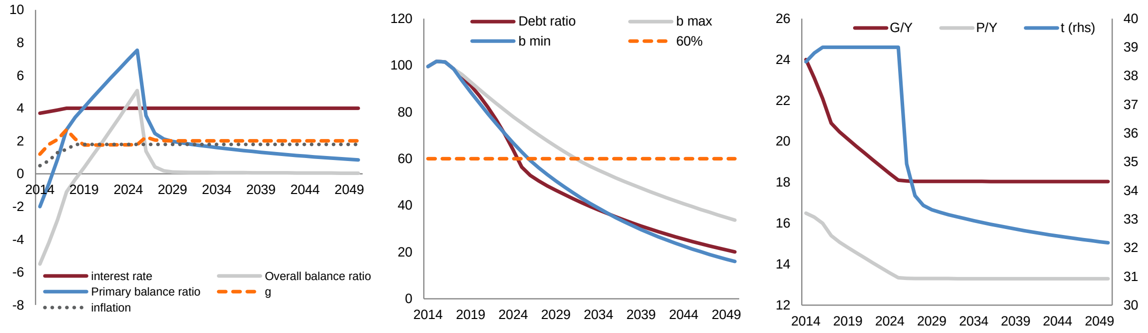
Semielasticities;n;tax					
T	G	I	rB	D	C
-0.01	-0.17	-0.21	-0.06	0.41	-0.20
0.01	0.99	0.99	0.93	0.66	0.30
Semielasticities;n;G					
T	G	I	rB	D	C
0.00	-0.07	-0.21	-0.06	0.32	-0.21
#	0.07	0.99	0.90	0.51	0.99
Semielasticities;n;I					
T	G	I	rB	D	C
0.00	-0.17	-0.23	-0.06	0.44	-0.23
#	0.99	0.41	0.95	0.73	0.41
Semielasticities;k;tax					
T	G	I	rB	D	C
-0.01	-0.17	-0.20	-0.06	0.41	-0.20
0.01	0.99	0.99	0.94	0.67	0.30
Semielasticities;k;G					
T	G	I	rB	D	C
0.00	-0.07	-0.20	-0.06	0.32	-0.20
#	0.07	0.99	0.90	0.51	0.99
Semielasticities;k;I					
T	G	I	rB	D	C
0.00	-0.17	-0.23	-0.06	0.44	-0.23
#	0.99	0.40	0.95	0.73	0.40

ANNEX 3. DEFAULT PROBABILITIES ESTIMATED FROM SIMULATIONS

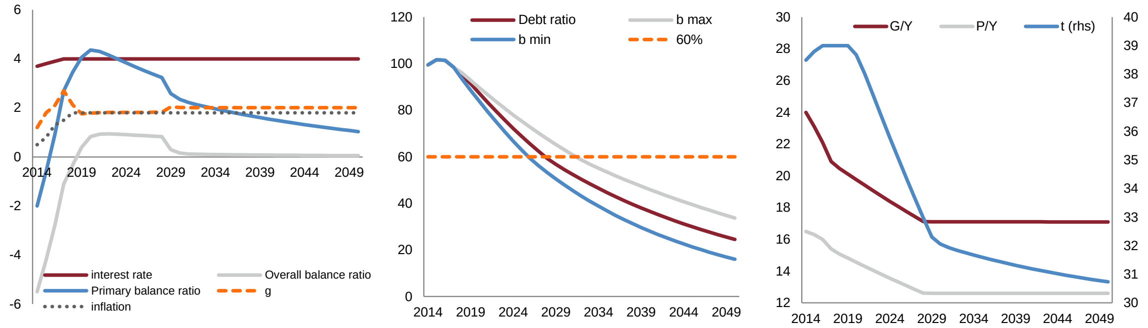
(u,v)= (0.05;0.5); 25 years, tax rate					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	0;0	0;0	0;0
60	0;0	0;0	0;0	0;0	0;0
80	0;0	0;0	0;0	1;1.0	3;1.7
100	0;0	0;0	0;0	27;4.4	58;4.9
120	0;0	4;1.9	22;4.1	57;4.9	99;1.0
(u,v)= (0.05;0.5); 25 years, G					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	0;0	0;0	0
60	0;0	0;0	0;0	0;0	3;1.7
80	0;0	0;0	0;0	7;2.6	20;4.0
100	0;0	0;0	5;2.2	43;4.9	64;4.8
120	0;0	7;2.6	53;5.0	89;3.1	99;1.0
(u,v)= (0.05;0.5); 25 years, I					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	0;0	0;0	0;0
60	0;0	0;0	0;0	0;0	0;0
80	0;0	0;0	0;0	0;0	0;0
100	0;0	0;0	0;0	13;3.4	42;4.9
120	0;0	0;0	12;3.25	92;2.7	100;0
(u,v)= (0.02;0.5); 25 years, tax rate					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	0;0	0;0	5;2.2
60	0;0	0;0	0;0	6;2.4	12;3.3
80	0;0	0;0	0;0	15;3.6	27;4.4
100	0;0	0;0	2;1.4	36;4.8	51;5.0
120	0;0	2;1.4	30;4.6	62;4.9	82;3.8
(u,v)= (0.02;0.5); 25 years, G					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	1;1.0	11;3.1	15;3.6
60	0;0	0;0	3;1.7	16;3.7	18;3.8
80	0;0	1;1.0	3;1.7	23;4.2	32;4.7
100	0;0	6;2.4	19;3.9	43;5.0	51;5.0
120	1;1.0	10;3.0	39;4.9	72;4.5	74;4.40
(u,v)= (0.02;0.5); 25 years, I					
b(0)=98.5	Interest rate reaction				
b*	0.01	0.03	0.05	0.10	0.15
40	0;0	0;0	0;0	0;0	0;0
60	0;0	0;0	0;0	0;0	2;1.4
80	0;0	0;0	0;0	6;2.4	16;3.7
100	0;0	0;0	3;1.7	30;4.6	55;5.0
120	0;0	2;1.4	23;4.2	70;4.58	81;3.92

ANNEX 4. CHARTS

Simulation 1



Simulation 2



Simulation 3